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4 HOURS

# Exam 8

## Advanced Ratemaking

### INSTRUCTIONS TO CANDIDATES

1. This 54.75 point examination consists of 23 problem and essay questions.
2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors.
  - Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.
  - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – DO NOT WRITE ON THE BACK OF THE PAPER. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.
  - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.
  - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.
3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.
4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

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- Verify that you have received the reference materials:
  - a. National Council on Compensation Insurance, Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2012 Study Kit*).
  - b. Insurance Services Office, Inc., Commercial General Liability Experience and Schedule Rating Plan.
  - c. National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2012 Study Kit*).
- 5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.
- 6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.
- 7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.
- 8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a copy of the examination from the CAS Web Site.

All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.
10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 15, 2012.

**END OF INSTRUCTIONS**

**EXAM 8 – FALL 2012**

1. (2.25 points)

Using Robertson's "NCCI's 2007 Hazard Group Mapping" as a case study, assess the extent to which the author's proposed hazard groups address any three key elements of the American Academy of Actuaries' "Risk Classification Statement of Principles."

**EXAM 8 – FALL 2012**

2. (2.25 points)

A private passenger auto insurance company orders a report whenever it writes a policy, showing what other insurance the policyholder has purchased. The following table shows claim frequencies (per 100 earned car-years) for bodily injury liability coverage, split by whether the policyholder has a homeowners policy and whether the policyholder had a prior auto policy:

Prior Auto Policy	Homeowners Policy	
	Yes	No
Yes	3	5
No	8	12

The table does not include the experience of policyholders with missing data.

a. (1.25 points)

Specify the following structural components of a generalized linear model that estimates frequencies for this book of business.

- i. Error distribution
- ii. Link function
- iii. Vector of responses
- iv. Vector of model parameters
- v. Design matrix

b. (1 point)

Describe how the missing data may cause problems for the company in developing the model, and suggest a solution.

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**EXAM 8 – FALL 2012**

3. (1.75 points)

The table below shows property claim frequency by year for the last five years. Assume that claim frequencies are Poisson distributed with a mean of 1.5.

Year	Exposures	Frequency
2011	118	1.5
2010	132	1.7
2009	121	1.3
2008	109	1.6
2007	97	1.3

The critical value for the relevant chi-squared distribution is 9.49.

a. (1.25 points)

Calculate the chi-squared test statistic for whether the claim frequency is shifting over time. Interpret the result.

b. (0.5 point)

Describe a second method for testing whether the claim frequency is shifting over time.

**EXAM 8 – FALL 2012**

4. (1.75 points)

An actuary has historical information relating to customer retention. A logistic model was used to estimate the probability of renewal for a given customer. The two variables determined to be significant were the size of rate change and number of phone calls the insured made to the company. The parameter estimates were determined to be as follows:

Rate Change	Parameter Estimate
Decrease to 3.9% increase	0.3323
4.0% to 6.9% increase	0
Increase of 7.0% or more	-0.4172

Number of Phone Calls in Past Year	Parameter Estimate
0	0
1	-0.2128
2+	-0.4239

Intercept Term	1.793
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a. (0.75 point)

Calculate the renewal probability for a customer who has a 7% rate increase and called the company twice in the past year.

b. (1 point)

The company needs policyholder retention to be above 78% to maintain growth and expense ratio goals. A possible strategy is to add the number of phone calls to the classification plan and use the model results to determine the rate increase.

Construct an argument either in favor of or against the strategy above, describing two reasons for that position.

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**EXAM 8 – FALL 2012**

5. (3 points)

The following data is used to price an excess of loss workers compensation policy:

- Data is available for the following injury types: fatal, permanent total injury (PT), major permanent partial (Major), minor permanent partial (Minor), temporary total (TT), and medical-only (Med).
- A multi-dimensional credibility technique (predicted) was used to estimate the frequency for class 5160.
- Class 5160 is in hazard group F.

Hazard Group F						
	Fatal	PT	Major	Minor	TT	Med
Frequency Relativity to TT	0.006	0.006	0.085	0.37	1.00	3.6
Severity Relativity to TT	80	100	30	4	1.00	0.3
Loss Elimination Ratio at \$250,000	27%	22%	57%	100%	100%	100%

TT Frequency per \$100 payroll	0.0002
TT Severity for Hazard Group F	\$10,000

Hazard Group F for Fatal Claims			
	Predicted	Raw Data	Holdout Sample
Quintile 1	0.75	0.70	0.90
Quintile 2	0.90	0.90	0.95
Quintile 3	1.00	1.00	1.00
Quintile 4	1.10	1.10	1.05
Quintile 5	1.25	1.30	1.10
Mean	1.00	1.00	1.00

Hazard Group F for PT Claims			
	Predicted	Raw Data	Holdout Sample
Quintile 1	0.70	0.75	0.80
Quintile 2	0.90	0.90	0.90
Quintile 3	1.00	1.00	1.00
Quintile 4	1.15	1.20	1.10
Quintile 5	1.20	1.25	1.20
Mean	1.00	1.00	1.00

- The hazard group relativities for Major, Minor, TT, and Med will be used.
- The multi-dimensional credibility relativities for PT claims will be used.
- Class 5160 is in Quintile 4 for both Fatal and PT claims.

a. (1.25 points)

Determine whether multi-dimensional credibility relativities should be used to estimate the expected loss for fatal claims.

b. (1.75 points)

Based on part a. above, calculate the expected loss for an excess of \$250,000 workers compensation policy with \$10 million in payroll.

**EXAM 8 – FALL 2012**

6. (2.5 points)

An insurance company has a private passenger auto book of business with the following claims experience:

Territory	Years Since Last Accident	Earned Premium at Present Rates for Two Years Since Last Accident	Earned Car Years	Number of Claims	Incurred Loss
1	0	\$15,000,000	15,000	5,000	\$9,000,000
1	1	\$125,000,000	125,000	41,000	\$75,000,000
1	2+	\$230,000,000	230,000	76,000	\$138,000,000
2	0	\$25,000,000	25,000	7,000	\$16,000,000
2	1	\$310,000,000	300,000	84,000	\$187,000,000
2	2+	\$550,000,000	535,000	147,000	\$328,000,000
3	0	\$10,000,000	10,000	4,000	\$7,000,000
3	1	\$80,000,000	100,000	35,000	\$43,000,000
3	2+	\$160,000,000	170,000	60,000	\$100,000,000

Choose an appropriate exposure base for calculating credibility. Justify the selection.

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**EXAM 8 – FALL 2012**

7. (2.5 points)

A reinsurer uses the following commission structure to pay a ceding company:

Provisional commission	30%	
Minimum commission	20%	At a 75% loss ratio
Sliding 1:1 to	35%	At a 60% loss ratio
Sliding 0.5:1 to a maximum	45%	At a 40% loss ratio

The aggregate loss distribution model is as follows:

Range of Loss Ratios	Average Loss Ratio in Range	Probability the Loss Ratio is in Range
0 – 40%	35%	0.04
40 – 60%	55%	0.32
60 – 75%	68%	0.24
75% +	80%	0.40

a. (1 point)

Calculate the expected loss ratio and expected sliding scale commission using the aggregate loss distribution model above.

b. (0.5 point)

Explain what a carryforward provision is and why it is used.

c. (0.5 point)

Assume the prior year's loss ratio was above 75%. Explain what effect a carryforward provision would have on the expected commission for the current year, all else being equal.

d. (0.5 point)

Briefly explain two approaches used in pricing the impact of a carryforward provision.

**EXAM 8 – FALL 2012**

8. (3 points)

An actuary decides to use the following exposure curve to price a risk and has determined that the appropriate  $b$  parameter is 0.15.

$$G(x) = \frac{1-b^x}{1-b}, \quad 0 \leq x \leq 1$$

a. (1.5 points)

Demonstrate that this function is a valid exposure curve.

b. (1 point)

Given that the maximum possible loss is \$2,000,000, use the selected exposure curve above to determine the ratio of pure risk premium in the layer \$1,000,000 excess of \$500,000.

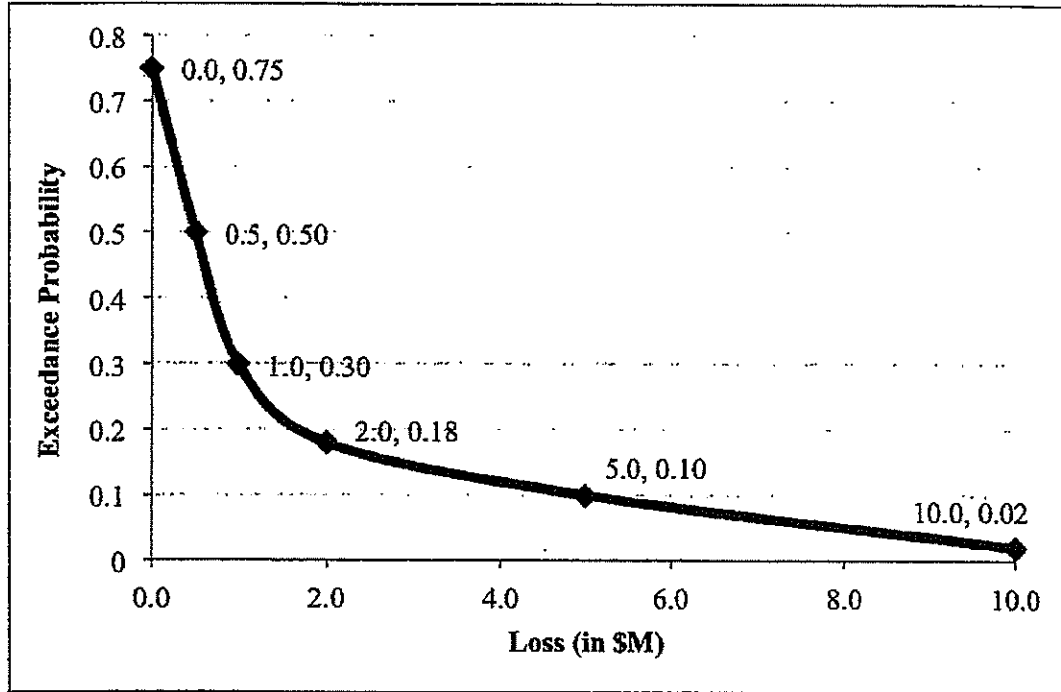
c. (0.5 point)

Discuss the appropriateness of the ratio of pure risk premium calculated above if the  $b$  parameter that the actuary selected was too high. State whether the actuary has underestimated or overestimated the probability of a total loss.

**EXAM 8 – FALL 2012**

9. (1.5 points)

The following exceedance probability curve is available for an insurer's portfolio:



a. (0.25 point)

Briefly explain what an exceedance probability curve represents.

b. (0.5 point)

The insurer wants to hold capital to support a 1 in 25 year Probable Maximum Loss (PML). Determine the loss level associated with this PML implied by the exceedance probability curve above.

c. (0.75 point)

Briefly discuss three common uses for exceedance probability curves.

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**EXAM 8 – FALL 2012**

10. (2 points)

An actuary has calculated an exposure curve using 10 years of the company's historical commercial property data, comprised of the following portfolio. All insured values used to calculate the exposure curve are on a per location basis.

Insured Value Range	Number of Risks	Total Premium	Pure Premium per Location
\$500,000 – \$1,000,000	415	\$23,810,000	\$34,539
\$1,000,001 – \$2,500,000	650	\$90,400,000	\$80,665
\$2,500,001 – \$5,000,000	350	\$92,970,000	\$172,659
\$5,000,001 – \$10,000,000	180	\$94,199,600	\$345,399

a. (1 point)

Evaluate the actuary's decision to use this book of business to produce an exposure curve.

b. (1 point)

The actuary is considering using the exposure curve to price a commercial property excess of loss treaty. The actuary is given the following information:

- A per risk limits profile for the subject commercial property business
- Ten years of historical ultimate loss ratios for the commercial property business:

Year	Ultimate Loss Ratio
2011	120%
2010	40%
2009	90%
2008	15%
2007	150%
2006	30%
2005	75%
2004	10%
2003	105%
2002	60%

Evaluate the actuary's decision to use the exposure curve to price the treaty.

**EXAM 8 – FALL 2012**

11. (2 points)

The fourth principle of Property and Casualty Insurance Ratemaking states:

A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer.

a. (1 point)

Defend the assertion that experience rating supports the principle that a rate should not be unfairly discriminatory.

b. (1 point)

Suppose the industry experience rating plan assigns too much credibility to individual experience for large insureds and assigns too little credibility to individual experience for small insureds. Argue that in a competitive insurance market, rates will not be unfairly discriminatory.

**EXAM 8 – FALL 2012**

12. (3.5 points)

Undeveloped losses follow a uniform distribution between \$0 and \$500. Each loss has an equal likelihood of developing such that it is multiplied by either 0.75 or 1.25.

a. (1 point)

Calculate the excess ratio at \$400 for undeveloped losses.

b. (1.5 points)

Calculate the excess ratio at \$300 for developed losses.

c. (1 point)

Suppose instead that loss multipliers are uniformly distributed between 0.75 and 1.25.

Determine whether the excess ratio at \$300 for developed losses will be higher than, equal to, or lower than the excess ratio calculated in part b above. Do not attempt to calculate the new excess ratio.

**EXAM 8 – FALL 2012**

13. (2 points)

The NCCI uses several procedures to add stability to workers compensation experience rating.

a. (1 point)

Using the NCCI methods as a model, propose two procedures to add stability to terrorism insurance ratemaking.

b. (1 point)

For each procedure in part a. above, provide one argument against including it in terrorism insurance ratemaking.

**EXAM 8 – FALL 2012**

14. (4 points)

The following information applies for a commercial general liability insured:

- All historical policies were effective January 1 to December 31.
- All historical policies were written on an occurrence basis.
- The policy effective January 1, 2013 will be on a claims made basis.
- The risk is products/completed operations only.
- The annual basic limit premium is \$100,000.
- The expected loss ratio is 70%.
- Experience is being evaluated as of June 30, 2012.
- A large loss is defined as \$250,000 or more in combined basic limit indemnity and ALAE.

Assuming all losses that occurred in the experience period meet the requirements to be defined as large losses, calculate the minimum number of large losses that must have occurred to trigger a debit modification for policy year 2013.



**EXAM 8 – FALL 2012**

15. (1.75 points)

An actuarial consulting firm is reviewing the inflation assumption used by a large insurer that writes casualty excess of loss coverage. The consulting firm has made the following assumptions regarding the insurer’s excess casualty book:

- Overall inflation is 8.0% and is assumed to have the same multiplicative effect on each size of loss.
- The unlimited, ground-up loss severity for the book of business follows a lognormal distribution with the expected loss equal to \$5,890,000.
- The following limited average severities, based on a lognormal distribution, apply to the insurer’s excess casualty book:

Per occurrence limit $k$	$E[g(x;k)]$	$E[g(x;k/1.08)]$
\$1,000,000	715,812	675,097
\$2,000,000	1,170,998	1,112,349
\$3,000,000	1,513,415	1,444,181
\$10,000,000	2,800,239	2,710,132
\$20,000,000	3,613,385	3,524,644
\$30,000,000	4,063,944	3,981,081
\$40,000,000	4,359,735	4,282,929
\$50,000,000	4,571,783	4,500,504

a. (1 point)

Using the consulting firm’s assumptions, calculate the average increase in excess losses due to inflation for a policy with a \$10,000,000 limit attaching at \$30,000,000.

b. (0.75 point)

The insurer agrees with the consulting firm’s overall trend assumption and general methodology, but believes that the average increase calculated in part a. above is too high. Describe any differences in assumptions the insurer may have with the consulting firm.

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**EXAM 8 – FALL 2012**

16. (2.5 points)

An actuary has experience rated five policies and presented the resulting modification factors to the underwriter. The results are as follows:

Policy	Experience Mod Factor	Manual Premium	Losses
A	0.97	\$40,000	\$39,000
B	1.40	\$10,000	\$14,500
C	0.95	\$25,000	\$23,500
D	1.33	\$15,000	\$20,500
E	0.81	\$45,000	\$33,000

a. (0.5 point)

The underwriter targets Policies B and D and states they should not be written because they are undesirable risks. Evaluate the validity of this statement.

b. (1 point)

Calculate the experience rating off-balance for these five risks.

c. (1 point)

Assess whether the plan used to calculate the experience modification factors demonstrates premium equity.

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**EXAM 8 – FALL 2012**

17. (2.5 points)

An actuary wants to calculate excess ratios for a book of business based on actual data below \$200,000, and from a mix of actual data and a fitted Pareto-exponential curve above \$200,000. The following data has been compiled:

Observed Excess Ratios by Hazard Group			
Accident Limit	HG A	HG B	HG C
\$50,000	0.42	0.46	0.58
\$100,000	0.21	0.26	0.38
\$150,000	0.11	0.16	0.26
\$200,000	0.06	0.11	0.18
\$250,000	0.04	0.08	0.13
\$300,000	0.03	0.06	0.10
\$500,000	0.01	0.03	0.05

Excess Ratios from Mixed Pareto-Exponential	
Entry Ratio	Excess Ratio
0.25	0.785
0.50	0.625
0.75	0.500
1.00	0.400
1.25	0.340
1.50	0.290
1.75	0.255
2.00	0.220

Average Size of Loss Truncated and Shifted to \$200,000	
HG A	\$75,000
HG B	\$100,000
HG C	\$200,000

a. (1.25 points)

Calculate the excess ratios for the following limits and hazard groups (HG):

- i. HG A; \$50,000
- ii. HG B; \$300,000

b. (0.75 point)

Briefly discuss three advantages of using this methodology to generate excess ratios.

c. (0.5 point)

Discuss whether it would be reasonable to fit a single curve rather than multiple curves by hazard group.

**EXAM 8 – FALL 2012**

18. (2 points)

The table below provides the actual loss history for 10 similar risks:

Risk	Sum of Losses Under \$200,000 per Accident	Individual Accidents Greater Than \$200,000	
		Accident 1	Accident 2
1	\$300,000		
2	\$400,000		
3	\$500,000		
4	\$600,000		
5	\$700,000		
6	\$800,000		
7	\$900,000		
8	\$1,000,000	\$300,000	
9	\$1,100,000	\$600,000	
10	\$1,200,000	\$200,000	\$1,400,000
Total	\$7,500,000	\$1,100,000	\$1,400,000

Construct Table L charges for a loss limit of \$500,000 at entry ratios of 1.10, 1.30, 1.60, 1.90 and 2.10.

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**EXAM 8 – FALL 2012**

19. (2.5 points)

The following applies to a retrospectively rated policy with a \$500,000 loss limitation. The insured has elected to include a premium element to stabilize premium adjustments.

Standard premium	\$2,000,000
Minimum premium	\$1,000,000
Maximum premium	\$3,000,000
Expected loss	\$1,400,000
Actual unlimited losses at second adjustment	\$1,700,000
Actual limited losses at second adjustment	\$1,500,000
Excess loss factor	0.15
Loss conversion factor	1.10
Retrospective development factor at second adjustment	0.07
Provision for expenses and profit exclusive of taxes	\$400,000
Converted insurance charge	\$100,000
Tax factor	1.05

a. (2 points)

Calculate the policy's retrospective premium at second adjustment.

b. (0.5 point)

Describe one reason an insurance company would include retrospective development factors in a policy.

**EXAM 8 – FALL 2012**

20. (3 points)

An Alaska workers compensation insured has standard premium of \$2,000,000. The insured is interested in a large dollar deductible policy with a deductible of \$225,000 and an aggregate of \$2,850,000.

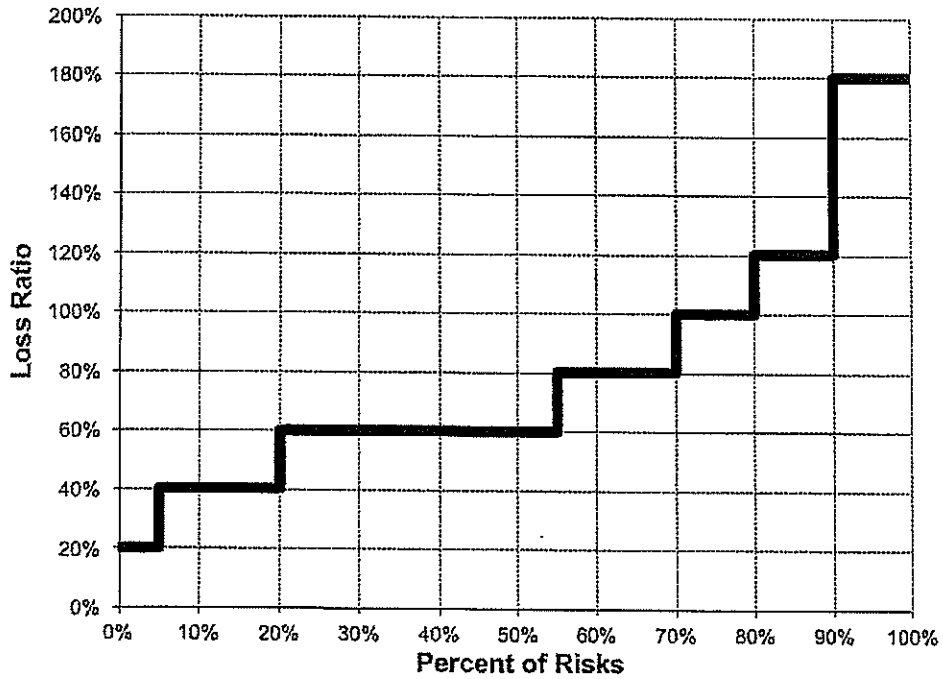
Assume the expected unlimited loss and ALAE ratio is 75%, and the risk is in hazard group B.

Calculate the total expected loss and ALAE for the large dollar deductible policy.

EXAM 8 – FALL 2012

21. (2 points)

A group of similar risks has an average loss ratio of 80%. The following Lee diagram depicts this group of risks:



a. (1.5 points)

Calculate the insurance savings at an entry ratio of 1.125.

b. (0.5 point)

The risks above the 90<sup>th</sup> percentile have their losses restated, significantly increasing the loss ratio. Describe the change to the insurance savings at an entry ratio of 1.125.

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**EXAM 8 – FALL 2012**

22. (3 points)

The current deductible pricing for an auto insurer is based on the following claim distribution:

Size of Loss	Number of Claims
\$100	21
\$250	50
\$500	42
\$1,000	37
\$5,000	22

An actuary wants to review the effect of loss trend on the insurer's loss elimination ratios.

a. (1 point)

Calculate the loss elimination ratio for a straight \$500 deductible assuming no trend adjustment.

b. (1.5 points)

Assuming no frequency trend, calculate the percentage change in the loss elimination ratio for a straight \$500 deductible assuming a ground-up loss severity trend of 10%.

c. (0.5 point)

Explain why the loss cost for a given straight deductible policy can increase by more than the ground-up severity trend.

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**EXAM 8 – FALL 2012**

23. (1.5 points)

The following information is available for a retrospective rating plan:

Expected total loss	\$100,000
Maximum loss	\$200,000
Minimum loss	\$50,000
Expense and profit provision (excluding taxes)	\$20,000
Loss conversion factor	1.35

The following is a Table M, for policies of this size:

Entry Ratio	Charge
0.25	0.85
0.50	0.76
0.75	0.69
1.00	0.64
1.25	0.59
1.50	0.56
1.75	0.52
2.00	0.49
2.25	0.46
2.50	0.44
2.75	0.42
3.00	0.40

Calculate the basic premium for this policy.

## Exam 8

QUESTION	POINT VALUE OF QUESTIONS	(a)	(b)	(c)	(d)	(e)	(f)	(g)
1	2.25							
2	2.25	1.25	1					
3	1.75	1.25	.5					
4	1.75	.75	1					
5	3	1.25	1.75					
6	2.5							
7	2.5	1	.5	.5	.5			
8	3	1.5	1	.5				
9	1.5	.25	.5	.75				
10	2	1	1					
11	2	1	1					
12	3.5	1	1.5	1				
13	2	1	1					
14	4							
15	1.75	1	.75					
16	2.5	.5	1	1				
17	2.5	1.25	.75	.5				
18	2							
19	2.5	2	.5					
20	3							
21	2	1.5	.5					
22	3	1	1.5	.5				
23	1.5							
TOTAL	54.75							

**Exam 8**  
**October 2012**

**Examiners' Report with  
Sample Solutions**

***As a result of Hurricane Sandy, some candidates sat for their exam on a make-up date established by the CAS office. The exam committee graded these candidates' answer sheets along with the other candidates' answer sheets, using the same administrative procedures and the same grading rubrics. The results of these candidates were then analyzed separately from the other candidates and no statistically significant difference was found between the two cohorts. Consequently, for each affected exam, the same pass mark was used for all candidates.***

## **Question 1:**

### ***Model Solution 1:***

New HG reflects the 3 statistical considerations of the AAA

- 1) Credibility: credibility assigned  $z = \max(1, [n/(n+k)] * 1.5)$  give larger classes more weight to permit the calculation of more accurate predictor of XS ratios.
- 2) Homogeneity: clustering analysis using k-means algorithm is used to assign each class into a hazard group. k-means has the property to minimize within variance and maximize between variance so new hazard groups are homogeneous.
- 3) Predictive Stability: since current HG were used as complement of credibility, this resulted in new HG where classes were assigned to same HG or to a lower HG in most cases. And since new HG were created based on class excess ratios, they reflect changes in insurance industry while they are stable to avoid premium swings.

### ***Model Solution 2:***

A risk classification should

- 1) Protect the insurance company's financial soundness: Robertson used a clustering method to group the ELF into their most appropriate HGs, this helped the insurers better identify the risk potential for each HG and quantify the potential losses more accurately and hence helped protect the financial soundness.
- 2) Ensure fairness: in the clustering method, the ELFs has been assigned into the HGs with respect to the closest centroid of each HG, this hence reduced the error in HG mapping. Also the model was sent to the underwriters for their opinions and some ELFs actually moved their HGs. All of this has ensured the HG development was built on scientific and objective criteria and this ensured fairness.
- 3) Encourage availability through financial incentive: In the HG mapping, the HGs were eventually grouped into 7 with the between variance between the groups maximized and the within variance minimized. Comparing with the historical 4 groups, the refined 7 group enhanced the classification and the insurer are hence more willing the provide broad range of coverages, since their financial situation are more secured with the new 7 HG system.

### ***Model Solution 3:***

- 1) Reflect expected cost differences as they are based on excess loss factors.
- 2) Applied objectively as the groups were assigned using the k-means algorithm.

3) Practical and cost effective as they keep the Hazard groups between 4 and 9 to keep a single digit code so the insurers wouldn't have to change their systems.

#### ***Model Solution 4:***

The 3 key elements I will consider are:

- 1) underwriting
- 2) statistical considerations
- 3) operational consideration

(all should be considered in designing a risk classification system)

1) underwriting - after the new 7 hazard groups were initially proposed, the NCCI asked underwriters to examine the proposed hazard groups and assess both mathematically and practical reasonability. This is important since if the classification system is misused in underwriting, results can be different than intended. Underwriters examined the groups and proposed moving more than 100 classes to different hazard groups, mainly for issues like two classes were so similar they shouldn't be in different hazard groups.

2) statistical - the classification system should meet the standards of homogeneity, credibility and predictive stability. The clustering algorithm served to minimize the within variance and maximize the between variance, tested by the Calinski-Harabasz statistic. This served to ensure homogeneity of the hazard groups. The new 7 hazard groups had a more even distribution of classes, ensuring hazard groups were large enough to be credible. The class excess ratio vectors were credibility weighted with the prior 4 group hazard group excess ratios, which served to ensure predictive stability and minimize swings.

3) operational - The NCCI knew that insurers would not be able to update their computer systems immediately to handle the new 7 group mapping, so they provided an interim mapping of the 4 group system to the 7 group system, so as not to cause undue expense in changing systems and to ensure a smooth transition.

#### ***Model Solution 5:***

1) Credibility - credibility was considered several times in Robertson's paper. When constructing the vectors of excess ratios for each class it was used to weight the class with the excess ratios in the current hazard group using the formula  $z = \min \left( \left[ \frac{n}{n+k} \right] * 1.5, 1 \right)$ . Credibility was again used in the clustering process as several data sets were run through the clustering algorithm depending on their credibility.

2) Constancy is an operational factor from the AAA. Constancy, should not see large shifts from classifications from one period to the next. Robertson considered this by reviewing the movement in classes between the new hazard groups under the various tests they performed using different credibility weights.

3) Public consideration - a risk should be able to identify with its classification. To consider this, Robertson utilized underwriters whom they provided with their initial results. Based off the feedback, some changes were made where classes were moved to certain hazard groups based on the other classes that a class may be aligned with and therefore closely related to from a public perception standpoint.

### **Model Solution 6**

Robertson's approach was to use vectors of credibility weighted excess ratios and cluster analysis to assign classes to Hazard Groups. The result of this was an increase in Hazard Groups from 4 to 7 HG and classes spread more evenly over the Hazard Groups.

Element 1: Avoid extreme discontinuities

By increasing the number of Hazard Groups, this reduces likelihood of significant changes in XS Loss factors / XS Ratios for being assigned to new hazard group. Less discontinuity.

Element 2: Homogeneity

Cluster Analysis and determining the optimal number of groups ensures that the risks in each class have similar XS loss potential while weighing credibility of data.

Element 3: Enhance Availability of Coverage by Allowing Economic Incentives to operate.

By better estimating class/HG XS ratios, Insurers will be more willing to offer coverage on higher excess ratio classes (rates will be adequate) and compete more (lower prices) on lower excess ratio classes.

### **Examiner's Comments**

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Question 1 asks that the candidate choose 3 elements from the Risk Classification Statement of Principles and show how Robertson's paper on NCCI's 2007 Hazard Group Mapping addresses those key elements. Thus, it was critical that the candidate be able to demonstrate how the NCCI analysis tied in with the elements or considerations found in the ASOP.

Candidates chose a variety of elements from the statement of principles which were accepted if explicitly found in the Risk Classification ASOP. Most candidates were able to identify the 3 elements. The grading committee was careful to determine whether, for each element identified, the candidate was able to show how it was addressed by the NCCI hazard group mapping analysis. As an example, if a candidate chose "homogeneity" as a key element, they would need to support this element by citing the hazard groups contained classes with similar *excess loss ratios* or that the

*cluster analysis* ensured that classes with similar excess loss potential were mapped into the same hazard groups. The underlying piece here was that the candidate cited something specific from the hazard group analysis which addressed homogeneity.

The most common error was the failure to support the element from the ASOP with specific ideas from the Robertson paper. A common example was a candidate response citing that the 3 key elements of a risk classification system were 1) Protect the insurer's financial soundness, 2) Enhanced fairness and 3) Economic incentive to promote widespread coverage availability. The candidate would go on to describe these concepts as is done in the ASOP, but did not cite anything from Robertson to show how these elements were addressed.

Another common mistake was simply stating that a given element of the ASOP was addressed because the number of hazard groups increased from 4 to 7. This response does not indicate that the candidate had any working knowledge of the 2007 hazard group analysis and therefore was considered unacceptable as supporting the element from the Statement of Principles.

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## Question 2:

### Model Solution 1

- a) i. Error should be Poisson for frequency.

$$P(x = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

- ii. Link function should be log link for multiplicative model.

$$g(x) = \ln(x)$$

$$g^{-1}(x) = e^x$$

iii. 
$$\begin{bmatrix} 3 \\ 5 \\ 8 \\ 12 \end{bmatrix}$$

iv. 
$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$
 where  $\beta_0$  is intercept

v. 
$$\begin{array}{cc} & x_1 & x_2 \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

$x_1$  is Prior Auto Policy = Yes

$x_2$  is Homeowners Policy = Yes

- b) Missing data can be problematic. If you put "unknown" as a level for each factor, for example, they will be perfectly correlated with each other. This will cause aliasing. To solve, you can eliminate the level from one of the factors so there are no linear dependencies. If there are linear dependencies, there will be no unique solution in beta parameters and any arbitrary amount can be added to one parameter and subtracted from the other.

### Model Solution 2

- a) i) Error distribution  $\rightarrow$  Poisson (since modeling claim frequencies)

- ii) Link fn  $\rightarrow$  Log link  $E(\underline{\gamma}) = \underline{\mu} \quad g^{-1}(\underline{\eta}) = e^{\underline{\eta}}$

iii)  $\underline{Y} = (3, 5, 8, 12)^\top$

iv)  $\underline{\beta} = (\beta_1, \beta_2, \beta_3)^\top$

$\rightarrow$  let  $x_1 = \begin{cases} 1 & \text{prior auto policy} = \text{Yes} \\ 0 & \text{otherwise} \end{cases}$

$x_2 = \begin{cases} 1 & \text{prior auto policy} = \text{No} \\ 0 & \text{otherwise} \end{cases}$

$x_3 = \begin{cases} 1 & \text{Homeowners policy} = \text{Yes} \\ 0 & \text{otherwise} \end{cases}$

v)  $X = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow$  ignoring policyholders with missing data

b) Missing data can lead to extrinsic aliasing. This occurs when there are linear dependencies in the observed data because of the nature of the data. In this case, the “missing” level for “prior auto policy” will be perfectly correlated with the “missing” level for “homeowners policy”. This can lead to convergence problems or confusing results. A solution would be to exclude these missing data records, or to reclassify them to an appropriate level.

**Examiner’s Comments**

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Part a

The most common errors were improperly identifying the error and link functions, or proposing a suboptimal alternative answer without proper justification. Answers containing design matrices that didn’t correctly correspond to the response vector or did not correct for aliasing were also encountered.

Generally, candidates did well with identifying the error distribution. Candidates who got the error distribution wrong either left it blank or picked a non-Poisson distribution without justifying that choice. Very few appeared to confuse the error distribution with the link function. The less-prepared candidate could usually guess at a distribution for the errors and would often name a distribution for the link function as well. Candidates who specified an incorrect function usually gave the identity function without justification.

Most candidates got at least two thirds of the vectors and design matrix correct. When they lost credit it was typically because they didn’t label the vectors clearly

enough. For example, a candidate might list all three without assigning subparts or labeling, or list a model vector without associating the betas with anything. Another common mistake was to specify a design matrix inconsistent with the vectors, usually because the matrix values were flipped between the  $y=5$  and  $y=8$  cases.

#### Part b

Most candidates got partial credit on this. By far the most typical mistake was an omission: Usually a candidate would either identify an explanation of how aliasing was a problem with the data, or how it would impact the model, but not both. Most candidates were able to present a reasonable solution or workaround for the problem. Some candidates lost credit by contradicting a correct statement, typically by implying that aliasing was a desirable characteristic of a model.



### Question 3:

#### Model Solution 1

a.

Year	Actual	Expected	$(A - E)^2 / E$
2011	$118 \times 1.5 = 177$	$118 \times 1.5 = 177$	0.00
2010	224.4	198	3.52
2009	157.3	181.5	3.23
2008	174.4	163.5	0.73
2007	126.1	145.5	2.59

$$\chi^2 = \sum \frac{(A - E)^2}{E}$$

$$= 10.06 > 9.49$$

⇒ Reject  $H_0$  and conclude parameter shift

- b. Correlation Test: Calculate the correlation between pairs of data for all years, group the correlations by number of years apart between data. If correlations decrease when the number of years apart increases, the claim frequency is shifting.

#### Model Solution 2

a.

$$\chi^2 = \sum \frac{w_i(O - E)^2}{E}$$

$$= 118 \frac{(1.5 - 1.5)^2}{1.5} + 132 \frac{(1.7 - 1.5)^2}{1.5} + 121 \frac{(1.3 - 1.5)^2}{1.5} + 109 \frac{(1.6 - 1.5)^2}{1.5} + 97 \frac{(1.3 - 1.5)^2}{1.5}$$

$$= 10.06$$

Since the observed  $\chi^2$  value is greater than the test statistic of 9.49, we can conclude that the claim frequency is shifting over time.

- b. Correlation of Years Test  
Create pairs of average frequency values, each separated by  $t$  years.  
Calculate correlation of values at each value of  $t$ .  
If correlation decreases for larger  $t$ 's, then we conclude frequency is shifting over time.

**Examiner's Comments:**

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Part a:

For full credit, a candidate had to correctly state and calculate the chi-squared test statistic, as well as correctly interpret this result relative to the given critical value and draw the correct conclusion regarding the parameter shift.

The most common error was not including exposures in the calculation of the chi-squared statistic. Other common errors included incorrectly stating the formula, or using an expected frequency other than the 1.5 given in the problem.

Part b:

Most candidates used the correlation of years test from Mahler, but other solutions were considered if they were well-described and justified.

\*\*\*\*\*

#### **Question 4:**

##### **Model Solution 1**

a)

$$E(Y_i) = g^{-1}(B_0 + B_1X_1 + B_2X_2)$$

Where  $B_0$  = intercept,

$X_1$  = rate change group: increase of 7%

$X_2$  = # phone calls: 2

For logistic model,  $g^{-1}(x) = e^x / (1 + e^x)$

$$E(Y) = g^{-1}(1.793 - 0.4172 - 0.4239)$$

$$E(Y) = g^{-1}(0.9519)$$

$$E(Y) = e^{0.9519} / (1 + e^{0.9519}) = .7215$$

b)

Against adding # phone calls to the class plan:

i.) It is easily manipulated by the insured

ii.) The variable lack constancy = the insureds # of phone calls might change dramatically year to year.

##### **Model Solution 2**

a)

$$\eta = \sum x\beta = 1.793 - 0.4172 - 0.4239 = 0.9519$$

Logistic model uses logit link, i.e.

$$\log\left(\frac{y}{1-y}\right) = \eta \quad \text{and } E(Y) = e^\eta / (1 + e^\eta) = e^{0.9519} / (1 + e^{0.9519}) = 0.7215$$

Renewal prob for the risk is 0.7215

b)

The strategy does not sound reasonable because:

- 1.) It is not based expected loss potential: insured who don't call the insurer may be those with clean history. They never find the need to make a phone call. While those who make phone calls may be those who had claims in the past. But insured can make phone calls for other purposes not related to claims.
- 2.) It is possible be manipulated by the insured: Once the insureds know making phone calls to the insurer can potentially alter the rate, they will either choose to call or NOT call depending what the impact is.

## Examiner's Comments

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### Part a

Most candidates were successful in calculating the systematic component, but struggled to remember the inverse link function for the logistic model. Candidates received partial credit if they assumed a different link function (and solved correctly). Candidates also received partial credit if they demonstrated knowledge of the logistic model (e.g. "the logistic model is a combination of the logit link function and a binomial error term") or by correctly identifying the link function or inverse link function; even if they did not solve the problem.

### Part b

Candidates struggled to get points on this question. Several candidates provided arguments both IN FAVOR and AGAINST the strategy, in which case credit was potentially given for the first position taken. Some candidates also provided more than two arguments IN FAVOR or AGAINST, in which case credit was potentially given for the first 2 arguments provided.

Otherwise, most well formulated arguments that relied on the considerations in designing a risk classification plan (AAA Risk Classification Statement of Principles) paper were given full credit, including but not limited to:

- 1.) Underwriting of individual risk is separate
- 2.) Marketing has an important influence on the mix of business attracted
- 3.) Program design - Degree of choice available to the buyer
- 4.) Statistical - Credibility, Predictive Stability:
- 5.) Operational - Expense: low as possible, minimize adverse selection, maximize equity; Constancy: variable should be constant in relation to the risk; Availability of Coverage; Avoidance of extreme ambiguities (collectively exhaustive and mutually exclusive); Manipulation: minimize ability to misrepresent; Measureability: moral character not determinable
- 6.) Causality: more acceptable to public if cause/effect relationship between risk and cost is demonstrable
- 7.) Controllability: risk control its own characteristics (good unless manipulation)

The ability for the insured to "manipulate the # calls" was a commonly chosen argument for which full credit was given, except in cases where the candidate argued that the insured would call less often.

\*\*\*\*\*

**Question 5:**

**Part a**

**Model Solution 1**

Calculate MSE on Predicted to Holdout for Fatal

$$.2 * ((.75-.9)^2 + (.9-.95)^2 + (1-1)^2 + (1.1-1.05)^2 + (1.25-1.1)^2) = .01$$

Calculate MSE on Hazard to Holdout for Fatal

$$.2 * ((1-.9)^2 + (1-.95)^2 + (1-1)^2 + (1-1.05)^2 + (1-1.1)^2) = .005$$

No reduction in MSE from multi-dimension credibility use Hazard Group

**Model Solution 2**

Calculate SSE on Predicted to Holdout for Fatal

$$((.75-.9)^2 + (.9-.95)^2 + (1-1)^2 + (1.1-1.05)^2 + (1.25-1.1)^2) = .05$$

Calculate SSE on Hazard to Holdout for Fatal

$$((1-.9)^2 + (1-.95)^2 + (1-1)^2 + (1-1.05)^2 + (1-1.1)^2) = .025$$

No reduction in SSE from multi-dimension credibility use Hazard Group

**Examiner's Comments:**

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Full credit was given to a candidate that also calculated MSE/SSE for Raw to Holdout if the two required calculations (Hazard to Holdout, Predicted to Holdout) were done.

Many candidates only did the Predicted to Holdout and Raw to Holdout calculation.

Some candidates used the PT data instead of Fatal.

A number of candidates identified the holdout group as the correct use but did not mention hazard grade, average, or 1.

\*\*\*\*\*

**Part b**

**Model Solution 1**

$$(\text{Frequency TT}) * (\text{Severity TT}) * \text{Payroll} * ((\text{Predicted Rel HG}) * (\text{Frequency Rel to TT}) * (1-\text{LER}) * (\text{Severity Rel to TT}))$$

$$\text{Fatal: } 1 * .006 * 80 * (1-.27) = .3504$$

$$\text{PT: } 1.15 * .006 * 100 * (1-.22) = .5382$$

$$\text{Major: } 1 * .085 * 30 * (1-.57) = 1.0965$$

$$\text{Total: } 1.9851$$



$$.0002 * 10,000 * (10,000,000/100) * 1.9851 = 397,020$$

**Model Solution 2**

(Expected Pure Premium) \* Payroll \* (XS Ratio)

$$\text{XS Ratio} = \frac{\text{SUM} ( (\text{Freq Rel}) * (\text{Sev Rel}) * (\text{Predicted Rel}) * (1-\text{LER}))}{\text{SUM}((\text{Freq Rel}) * (\text{Sev Rel}) * (\text{Predicted Rel}))}$$

Type	Freq Rel	Sev Rel	Predicted	(1 - LER)	Numerator	Denominator
Fatal	.006	80	1	.73	.3504	.48
PT	.006	100	1.15	.78	.5382	.69
Major	.085	30	1	.43	1.0965	2.55
Minor	.37	4	1	0	0	1.48
TT	1	1	1	0	0	1
Med	3.6	.3	1	0	0	1.08

$$= 1.9851 / 7.28 = 27.27\%$$

$$\text{Expected Pure Premium} = 7.28 * (.0002)(10,000) = 14.56$$

$$14.56 * 10,000,000/100 * .2727 = 397,051$$

**Model Solution 3**

(ELR) \* Payroll \* (XS Ratio)

$$\text{XS Ratio} = \frac{\text{SUM} ( (\text{Freq Rel}) * (\text{Sev Rel}) * (\text{Predicted Rel}) * (1-\text{LER}))}{\text{SUM}((\text{Freq Rel}) * (\text{Sev Rel}) * (\text{Predicted Rel}))}$$

Type	Freq Rel	Sev Rel	Predicted	(1 - LER)	Numerator	Denominator
Fatal	.006	80	1	.73	.3504	.48
PT	.006	100	1.15	.78	.5382	.69
Major	.085	30	1	.43	1.0965	2.55
Minor	.37	4	1	0	0	1.48
TT	1	1	1	0	0	1
Med	3.6	.3	1	0	0	1.08

$$= 1.9851 / 7.28 = 27.27\%$$

$$\text{ELR} = 1.43$$

$$1.43 * 10,000,000/100 * .2727 = 38,996$$

**Examiner's Comments:**

\*\*\*\*\*

Full credit was given for the three solutions noted above where the excess loss amount was calculated differently. Full credit was also given for an ELR of 1.53.

In model solution 3, a candidate could use the NCCI manual to come up with the expected loss per 1,000 for the class code even though that was not the intention of the problem. A candidate who used the given info to calculate an excess ratio and then used the NCCI ELR (instead of the given frequencies and severities) for the total loss received full credit. Only a handful of candidates successfully did this.

A fair amount of candidates came to the conclusion in Part A that the multi-dimension credibility should be used, but then did not include the Fatal Predicted relativity of 1.1 in their calculation.

Quite a few candidates did not include the PT Predicted adjustment of 1.15

A handful of candidates only completed the Fatal component of the answer above.

A fair amount of candidates did not properly calculate the XS ratio.

\*\*\*\*\*

**Question 6:**

**Model Solution 1**

Premium should be used as the base to prevent the maldistribution of premium IF higher frequency territories have higher premiums AND territory differentials are correct.

	(A)	(B)	(C)	(D)	(A)/(B)	
<u>Terr</u>	<u>EP</u>	<u>ECY</u>	<u># Clms</u>	<u>Losses</u>	<u>Avg EP</u>	<u>Rel EP</u>
1	370,000,000	370,000	122,000	222,000,000	1,000	1.0033
2	885,000,000	860,000	238,000	531,000,000	1,029	1.0325
3	<u>250,000,000</u>	<u>280,000</u>	<u>99,000</u>	<u>150,000,000</u>	<u>893</u>	<u>0.8958</u>
Total	1,505,000,000	1,510,000	459,000	903,000,000	997	1.0000

	(C)/(D)		(C)/(B)	
<u>Terr</u>	<u>Avg PP</u>	<u>Rel PP</u>	<u>Freq</u>	<u>Rel Freq</u>
1	1,820	0.9249	0.3297	1.0847
2	2,231	0.1341	0.2767	0.9104
3	<u>1,515</u>	<u>0.7702</u>	<u>0.3536</u>	<u>1.1632</u>
Total	1,967		0.3040	1.000

Compare Rel Freq to Rel EP. They are not lining up. Terr 3 has the highest rel freq but the lowest rel EP.

That means ECY is a more appropriate base than EP.

**Model Solution 2**

Bailey and Simon use EP as exposure base to eliminate maldistribution due to high frequency territories having high territorial relativity and a high # of non-accident-free risks.

Hazaam says this exposure base works only when:

- 1) High frequency territories are also high premium territories, and
- 2) Territorial relativities are proper

Thus, I will test the two above points.

1:	<u>Territory</u>	<u>Frequency (<math>\Sigma</math>claims/<math>\Sigma</math>car years)</u>	<u>avg Prem (<math>\Sigma</math>Prem/<math>\Sigma</math>car years)</u>
	1	.330	1000
	2	.277	1029
	3	.354	893

High frequency territories do not appear to have higher avg premium.

2:	<u>Territory</u>	<u>Loss</u>	<u>Prem</u>	<u>LR (Loss/Prem)</u>
	1	222M	370M	.6
	2	531M	885M	.6
	3	150M	250M	.6

All territories have the same LR, which suggests that the relativities are proper. However, due to #1 failing, I would not use EP, and instead use earned car years.

**Model Solution 3**

Check territory differentials

	Average Premium	Loss Ratio	Frequency
Terr 1 0	1000	60%	33%
1	1000	60%	33%
2+	1000	60%	33%
Terr 2 0	1000	64%	28%
1	1033	60%	28%
2+	1028	59.6%	27%
Terr 3 0	1000	70%	40%
1	800	54%	35%
2+	941	62.5%	35%

Bailey & Simon use EP @ current rates for exposure base to eliminate maldistribution of high frequency territories causing more 0, 1 risks  
 Hazaam points out this only works if high frequency territories are high premium and territory differentials are proper

In this case, territory 3 differential is not proper, and it has highest frequency without high premium.

∴ choose car years as exposure base for credibility

**Examiner’s Comments:**

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This question was somewhat polarizing. Many candidates responded well, and received a significant portion of the points. Many other candidates did not know how to approach the problem and received little or no points.

The most common mistake was approaching the question as a problem about how to calculate credibility but not answering anything about what exposure base should be used.

The next most common mistake was answering that one should use earned premium as the exposure base as stated in Bailey & Simon but not looking at whether the data met Hazaam’s conditions.

Another common mistake was providing the calculations needed to draw a proper conclusion but not providing a conclusion or providing the wrong conclusion. Another common mistake was not looking at average earned premium and frequency, but the candidate instead looked at total earned premium and total losses. They then draw the wrong conclusion.

\*\*\*\*\*

**Question 7:**

**Part a**  
**Model Solution 1**

LR Range	Commission at Ave LR in Range
0-40	.45
40-60	$.35 + [.5 * (.6 - .55)] = .375$
60-75	$.2 + (.75 - .68) = .27$
75+	.2

Expected Loss Ratio =  $(.35 * .04) + (.55 * .32) + (.68 * .24) + (.8 * .4) = .6732$

Expected Commission =  $.45 * .04 + .375 * .32 + .27 * .24 + .2 * .4 = .2828$

**Examiner's Comments:**

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Candidates generally performed well on this part. Two common mistakes were calculation of commission using the sliding scale or forgetting to calculate the expected loss ratio.

\*\*\*\*\*

**Part b**  
**Model Solution 1**

States that if the past years' loss ratio(s) have been greater than the loss ratio yielding the minimum commission, then the amount by which the loss ratios exceeded that loss ratio (75% in this example) shall be added to the current year's loss ratio in calculating the sliding scale commission

Used to help stabilize the reinsurers' results.

**Model Solution 2**

If the insurer's loss ratio is above the loss ratio corresponding to the minimum commission, the difference between the two loss ratios is "carried forward" to the next year for the purposes of calculating next year's commission. Sometimes also done for loss ratios below maximum commission loss ratio. This is done to smooth out results and encourage insurer to control losses.

**Model Solution 3**

A carryforward allows the losses over the maximum loss ratio to be carried over to the next year for that year's calculation of the commission. This gives insurers incentives to control losses once their loss ratio passes the maximum threshold.

**Examiner's Comments:**

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Common mistakes included candidates forgetting to answer the “why is it used” part of the question and failure to clearly describe the carryforward provision in terms of the “EXCESS” losses being transferred to the subsequent year.

\*\*\*\*\*

**Part c**

**Model Solution 1**

Since the amount by which the prior year loss ratio exceeds 75% is added to the current year loss ratio, that will decrease the expected commission.

**Model Solution 2**

If the loss ratio was greater than 75%, the amount beyond that would be added to current year's loss ratio and hence, this year's adjusted loss ratio would be higher and thus the ceding commission lower.

**Model Solution 3**

You shift the scale by the amount of the carryforward, thus the loss ratio ranges would decrease since the prior year loss ratio was too high. This will have the effect of lowering the commission ratio in the current period. So for example, may need to get 30% loss ratio to get the maximum commission instead of 40%.

**Examiner's Comments:**

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Common mistakes included candidates forgetting to mention in their answer that the commission would decrease and candidates comparing the actual loss ratio to the expected loss ratio instead of the loss ratio at the minimum commission.

\*\*\*\*\*

**Part d**

**Model Solution 1**

1- Calculate the impact on the current year only (ignore that it can continue to be carried forward in more and more years). Shift the scale and aggregate loss ratio ranges by the amount of carryforward to the current year.

2- Look at the longer term impact by modeling a block of years rather than just one year.

**Model Solution 2**

Can estimate impact on a single year by sliding the ranges to recognize the additional loss ratio → slide ranges of ceding commission and ranges of buckets for aggregate distribution → but this ignores any potential carryover that could go for multiple years ( if multiple –year carryforward included in treaty)

-or-

Can estimate increased stability on carryforward over multiple years by pricing based on the expected loss ratio distribution over multiple years → but how to reflect stabilizing of aggregate distribution is not obvious and what if policy is cancelled.

**Model Solution 3**

Add effect of carryforward to current year only  
Model the long-term loss ratio distribution instead of current year distribution.

**Examiner’s Comments:**

\*\*\*\*\*  
Candidates generally performed worse on this subpart when compared to other parts of the questions. Common mistakes included generic appeals to simulation and modeling (without follow-through) and the discussion spreading the carryover over multiple years.  
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### Question 8:

#### Part a

#### Model Solution 1

a)  $G(x)$  should be increasing and concave and have interval  $[0,1]$

$$G(0) = \frac{1-b^0}{1-b} = 0 \quad G(1) = \frac{1-b^1}{1-b} = 1$$

$$G'(x) = \frac{-(\ln(b))b^x}{1-b} = \frac{-(\ln(0.15))b^x}{1-0.15} > 0$$

$$G''(x) = \frac{-(\ln(b)^2)b^x}{1-b} < 0$$

Therefore this function is a valid exposure curve.

#### Model Solution 2

a) This function is a monotonically increasing function of  $x$ .

$$G(1) = \frac{1-0.15}{1-0.15} = 1 \quad G(0)=0, \quad 0 \leq G(x) \leq 1 \text{ since } 0 \leq x \leq 1$$

It is also a member of the MBBEFD family of curves where  $gb=1$  and  $g>0$ . These are valid exposure curves, so  $G(x)$  must be valid as well.

#### Model Solution 3

$$\boxed{G(0) = 0} \rightarrow G(0) = \frac{1-b^0}{1-b} = \frac{0}{1-b} = 0$$

$$\boxed{G(1) = 1} \rightarrow G(1) = \frac{1-b^1}{1-b} = \frac{1-b}{1-b} = 1$$

$$\text{Also } \boxed{G'(0) \geq 1 \geq G'(1) \geq 0}$$

$$G'(0) = \frac{-(\ln(0.15))0.15^0}{1-0.15} = 2.232$$

$$G'(1) = \frac{-(\ln(0.15))0.15^1}{1-0.15} = 0.335$$

$$\boxed{2.232 \geq 1 \geq 0.335 \geq 0}$$

Exposure curve is concave and increasing.

#### Model Solution 4

$$a) G(0) = \frac{1-0.15^0}{1-0.15} = \frac{0}{0.85} = 0$$



$$G(1) = \frac{1 - 0.15^1}{1 - 0.15} = \frac{0.85}{0.85} = 1$$

$$\mu = \frac{1}{G'(0)} = \frac{1}{2.232} = 0.448$$

$$p = \frac{G'(1)}{G'(0)} = \frac{0.3348}{2.232} = 0.15$$

$0 \leq p \leq \mu \leq 1$   
 $0 \leq 0.15 \leq 0.448 \leq 1$  so it is a valid exposure curve.

**Examiner's Comments:**

\*\*\*\*\*

Common mistakes were to state the criteria for an exposure curve without demonstrating that the given formula met these criteria and was a valid exposure curve.

Another acceptable approach that some candidates used to prove either the increasing or concave criteria was to graph the function by plugging in points.

\*\*\*\*\*

**Part b**

**Model Solution 1**

$$b) \quad x_1 = \frac{0.5M}{2M} = 0.25 \quad x_2 = \frac{1.5M}{2M} = 0.75$$

$$G(x_1) = \frac{1 - 0.15^{0.25}}{1 - 0.15} = 0.4443$$

$$G(x_2) = \frac{1 - 0.15^{0.75}}{1 - 0.15} = 0.8929$$

Therefore the ratio of pure premium is  $0.8929 - 0.4443 = \underline{0.4486}$

**Examiner's Comments:**

\*\*\*\*\*

A common mistake was to either normalize the layer incorrectly or to use the wrong layer for one of the calculations above. The most common wrong layer was to use  $1M/2M$  instead of  $1.5M/2M$ .

\*\*\*\*\*

**Part c**

**Model Solution 1**

c) If the b parameter that the actuary selected was too high, the ratio of pure risk premium above will increase. Because  $G(x)$  is a concave and increasing function if b parameter is too high the actuary has overestimated the probability

of a total loss. Because the probability of total loss is  $G'(1)/G'(0)$ , and  $G'(1)/G'(0) = b$ . So when the  $b$  parameter was too high the probability of total loss has been overestimated.

**Model Solution 2**

c) Since probability of total loss =  $b$ , when  $b$  is too high the probability of total loss is too high.

$$p = \frac{G'(1)}{G'(0)} = \left( \frac{-(\ln(b))b^1}{1-b} \right) / \left( \frac{-(\ln(b))b^0}{1-b} \right) = b$$

**Model Solution 3**

c)  $bg = 1$  if  $b$  is too high, then  $g$  was too low.  $\text{Pr}(\text{total loss}) = p = 1/g$  so probability of total loss was too high.

**Model Solution 4**

c) If  $b = 0.1$  then  $G(0.75) = 0.913$  and  $G(0.25) = .486$ ,  $G(0.75) - G(0.25) = 0.427$ . If  $b$  is too high then the ratio of pure risk premium in the layer above is too high. Thus the probability of total loss is overestimated.

**Examiner's Comments:**

\*\*\*\*\*

There were several ways to make a connection between the  $b$  parameter and the probability of total loss. The ideal approach was to show that  $b = p$ ; although the candidate did not need to know this to earn full credit as shown in the samples above.

Another acceptable approach was to discuss the changing shape of the exposure curve as the  $b$  parameter changed; and how this related to the probability of total loss.

The most common mistake in this section was to essentially prove that the layer of loss in part b was overstated without ever making the connection to, or mentioning, how the probability of total loss is impacted.

\*\*\*\*\*

**Question 9:**

**Part a**

**Model Solution 1**

An exceedance probability curve shows all possible levels of annual loss and the annual probability that the given loss level is exceeded.

**Model Solution 2**

The probability for a loss to exceed a certain level given a period of time.

**Examiner's Comments:**

\*\*\*\*\*

Most candidates did well on this part and got full credit. The candidates that did not get full credit mostly explained that the graph was the probability of a particular outcome, not exceeding the outcome.

\*\*\*\*\*

**Part b**

**Model Solution 1**

$$1/25 = .04 \Rightarrow PML = 10M(0.04-0.02)/(0.1-0.02)*5M = 8.75M$$

**Examiner's Comments:**

\*\*\*\*\*

To get full credit, the candidate needed to understand 1/25 years is a probability of 0.04 and that they needed to linearly interpolate and get 8.75M. A common error made by candidates was to eyeball the graph and select an amount, rather than complete the interpolation and show their work. Other errors included incorrectly interpolating.

Also some candidates incorrectly assumed the curve was discreet and either selected 5M or 10M.

\*\*\*\*\*

**Part c**

**Model Solutions**

- 1) Calculate the PML for a given payout period
- 2) Calculate if the portfolio meets solvency goal
- 3) Decide how much proportion of the risks should be ceded
- 4) Used to calculate average annual loss
- 5) Set level of conservativeness like PML in 1/X chance

- 6) Used to find a strategy to change the portfolio if it is currently above the level of conservativeness
- 7) Used by emergency response unit to determine where damage might be and build strategies in times of catastrophes
- 8) Used when running logic trees. Instead of using point estimates, each branch of tree can have its own exceedance prob curve for the different outcomes. Then can combine.
- 9) Emergency management services: to evaluate potential risk of some regions and plan evacuation.
- 10) Reinsurance Broker: to evaluate the level of risk of their portfolio and estimate the impact of accepting new risks.

**Examiner's Comments:**

\*\*\*\*\*

Most answers having to do with solvency, AAL, PML, portfolio management, cat modeling, pricing insurance / reinsurance, reinsurance layers, etc. got full credit. Candidates that did not get full credit said the same answer twice in a different way eg. Risk management and management of risk...

In addition, if candidates provided more than three responses, only the first three were reviewed for grading.

Examples of responses that did not receive credit are:

- Reserving
- Academic Research
- To see if portfolios are correlated
- Exposure distribution/exposure rating/exposure curve
- To estimate physical damage to property

\*\*\*\*\*

## **Question 10:**

### **Part a**

#### **Model Solution 1**

IV Midpoint	Pure Premium	PP/ IV
750K	34,539	0.04506
1,750K	80,665	0.04609
3,750K	172,659	0.04604
7,500K	345,399	0.04605

Using the same exposure curve for multiple insured values implies that the likelihood of loss as a percentage of insured value is the same across all insured values (i.e. scale independence). The table above shows that the pure premium per location as a percentage of insured value is consistent across the various insured values. This indicates that the actuary's decision to create an exposure curve using the combined data is reasonable.

#### **Model Solution 2**

Clark states this is not as ideal for a commercial book as for homeowners. The reason is that homeowner risks are very homogeneous, even over a large size of home range. Commercial risks are very heterogeneous in terms of both construction type and occupancy (which affects contents), therefore over the large spectrum of size of risks, it is difficult to tell if a 10% loss is as likely to a \$1m property as to a \$10m one.

#### **Model Solution 3**

Yes this is a good set of data to use because one assumption underlying an exposure curve is that the size of risk is irrelevant. With this book of business the pure premium for 500,000 – 1,000,000 is 10% of the pure premium for 5,000,000 – 10,000,000 and  $500,000 / 5,000,000 = 10\%$ . The same goes for the other insured value ranges. Therefore this curve will be appropriate to use for any size risk and is a good curve.

### **Part b**

#### **Model Solution 1**

I disagree. This is a very volatile book of business. The loss ratio may be 10% some years and 150% others. I wouldn't use the exposure curve generated by the book of business to price it without better understanding the loss ratio volatility. Also, the profile is given "per risk" instead of per location, which will skew results because different risks may have different numbers of locations.

**Model Solution 2**

The profile given is on a per risk level, while the exposure curve was calculated based on a per location level. Also, the historical loss ratios are so volatile that it's difficult to determine an expected loss ratio for the rating period. The actuary's decision is not sound based on this information.

**Examiner's Comments:**

\*\*\*\*\*

Part a

The examiners are looking for the demonstration and/or discussion of scale independence and its criticality in the development of exposure curves. Common responses that, by themselves, did not receive credit include:

- Loss ratio comparisons between ranges
- Judgment of credibility based on the number of risks and/or premiums in each range
- Separation of ranges for arbitrary reasons/analysis (e.g. assumptions of free cover by range)

In addition, candidates did not receive full credit if they did not opine on the actuary's decision.

Part b

Responses that did not receive credit:

- Did not address one of the two pieces of information given (per risk limits profile and loss ratio history)
- Due to experience fluctuation agreed with actuary's decision to use it
- Comparison of historical average loss ratio to an average loss ratio calculated in part a

In addition, candidates did not receive full credit if they did not opine on the actuary's decision.

\*\*\*\*\*

**Question 11:**

***Model Solution 1***

Part a

The main goal of experience rating is individual risk equity. Experience rating recognizes that each risk has a different loss potential, so by modifying rate appropriately, the expected profit potential for each risk can be made equal. This ensures that equity is achieved and rates are not unfairly discriminatory. Otherwise, risks with lower loss potential within the class would subsidize other higher loss potential risks.

Part b

Large risks with credit mods will have worse experience than expected. Similarly, small risks with credit will have better experience than expected. Forces of supply and demand in competitive market will push rates down for large debit and small credit risks. All risks now pay rates commensurate for their exposure to loss so rates are not unfairly discriminatory.

**Examiner's Comments:**

\*\*\*\*\*

Part a

Responses that defined experience rating rather than explaining how experience rating is not unfairly discriminatory did not receive credit. As an example, mentioning that experience rating uses historical losses did not receive credit but those who explained those historical losses are predictive of future loss potential did.

Part b

Responses that explained what a competitive market is did not receive credit but those that explained how insureds would react in a competitive market did. Stating that adverse selection would result did not receive credit as it misses how insureds would compete for preferred business. Candidates were also expected to mention how large and small insureds would perform given the incorrect credibility.

\*\*\*\*\*

**Question 12:**

**Model Solution 1**

a.

$$R(400) = \frac{\int_{400}^{500} (x - 400) \left(\frac{1}{500}\right) dx}{E[X]}$$

$$= \frac{10}{250} = \underline{\underline{0.04}}$$

$$E[X] = \frac{0 + 500}{2} = 250$$

$$\int_{400}^{500} (x - 400) \left(\frac{1}{500}\right) dx = \frac{1}{500} \left(\frac{x^2}{2} - 400x\right) \Big|_{400}^{500} = 10$$

b.

$$\hat{R}(300) = (1/2)(0.75)R(300/.75) + (1/2)(1.25)R(300/1.25)$$

$$= (1/2)(.75R(400) + 1.25R(240))$$

$$= (1/2)(.75 \cdot .04 + 1.25 \cdot .2704)$$

$$= \underline{\underline{0.184}}$$

$$\int_{240}^{500} (x - 240) \left(\frac{1}{500}\right) dx = \frac{1}{500} \left(\frac{x^2}{2} - 240x\right) \Big|_{240}^{500}$$

$$= 67.6$$

$$R(240) = \frac{67.6}{250} = 0.2704$$

c.

Old multipliers:  $E[X] = .5(.75) + .5(1.25) = 1$

$$E[X^2] = .5(.75^2) + .5(1.25^2) = 1.0625$$

$$\text{Var}(X) = 1.0625 - 1^2 = .0625$$

$$\mu = 1 \quad \sigma = \sqrt{.0625}$$

$$\text{Coefficient of Variation} = \frac{\sqrt{.0625}}{1} = 0.25$$

New multipliers

$$E[X] = \frac{.75 + 1.25}{2} = 1$$

$$E[X^2] = \int_{.75}^{1.25} x^2 \left(\frac{1}{1.25 - .75}\right) dx = \frac{1}{0.5} \left(\frac{x^3}{3}\right) \Big|_{.75}^{1.25} = 1.0208$$

$$\text{Var}(X) = 1.0208 - 1^2 = 0.0208$$

$$\mu = 1 \quad \sigma = \sqrt{.0208}$$

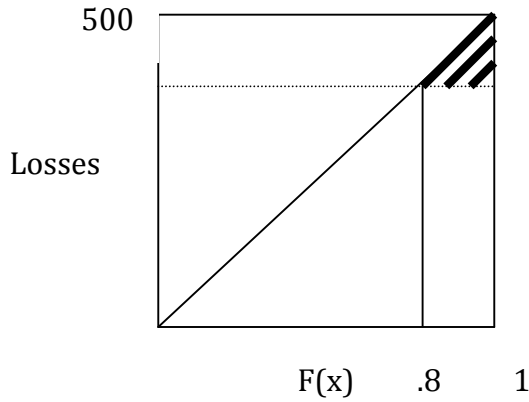
$$\text{Coeff of Variation} = \frac{\sqrt{.0208}}{1} = 0.144$$

Since the coefficient of variation for the loss multipliers from 0.75 to 1.25 based on the uniform distribution is smaller, the impact of dispersion will be less & thus the excess ratio will be lower.



**Model Solution 2 (parts a and b only)**

a.



$R(400) = \text{shaded area} / \text{total area}$

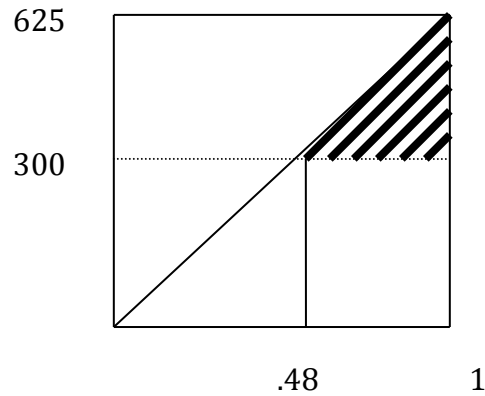
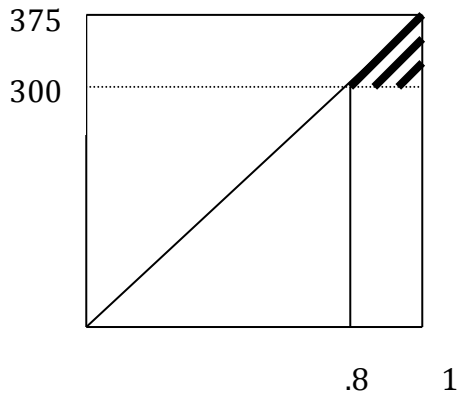
$$= \frac{\left(\frac{1}{2}\right)(.2)(100)}{\left(\frac{1}{2}\right)(1)(500)}$$

$$R(400) = .04$$

b.

Developed, .75

Developed, 1.25



$$R(300) = \frac{\left(\frac{1}{2}\right)(.2)(75)}{\left(\frac{1}{2}\right)(1)(375)} = .04$$

$$R(300) = \frac{\left(\frac{1}{2}\right)(.52)(325)}{\left(\frac{1}{2}\right)(1)(625)} = .2704$$

$$\hat{R} = \frac{\left(\frac{1}{2}\right)(187.5)(.04) + \left(\frac{1}{2}\right)(312.5)(.2704)}{\left(\frac{1}{2}\right)(187.5) + \left(\frac{1}{2}\right)(312.5)} = .184$$

**Model Solution 3 (part b only)**

After development, we have two uniform distributions with 50%, 50% chance. One is between 0 & 375 i); the other is 0 & 625 ii)

$$\text{uniform i) } \int_{300}^{375} G(x)dx = \int_{300}^{375} \frac{375-x}{375} dx = 75 - \frac{x^2}{750} \Big|_{300}^{375} = 7.5$$

$$\text{uniform ii) } \int_{300}^{625} G(x)dx = \int_{300}^{625} \frac{625-x}{625} dx = 325 - \frac{x^2}{1250} \Big|_{300}^{625} = 84.5$$

$$\text{uniform i) Average loss} = \frac{0 + 375}{2} = 187.5$$

$$\text{uniform ii) Average loss} = \frac{0 + 625}{2} = 312.5$$

$$\text{Excess ratio } R(300) = \frac{7.5 * 0.5 + 84.5 * 0.5}{187.5 * 0.5 + 312.5 * 0.5} = 18.4\%$$

**Model Solution 4 (part c only)**

It will be lower, essentially because the uniform distribution will be less dispersed vs the distribution above. (The dist above can only be either endpoints => greater deviation from the mean vs a uniform that gives weight to intermediate values).

Decreasing the CV ( $= \frac{\sigma}{\mu}$  = measure of dispersion) will decrease the XS ratio at higher limits like 300.

**Examiner's Comments:**

\*\*\*\*\*

Part a was very straightforward and most candidates did very well.

On part b, most candidates did well and utilized the developed loss excess ratio formula correctly. The most common mistakes were:

- Confusing multipliers with divisors;
- Adding an extra term in the formula to account for no development (which was irrelevant to the problem); and
- Simply averaging R(400) and R(240) instead of weighting them by the multipliers

On part c, in order to get full credit, the candidate needed to calculate the coefficient of variation (CV) of each distribution or explain in detail why the CV of the uniform distribution would be less than that of the discrete distribution (and therefore that the excess ratio would be less). Most candidates did not mention the relationship that the excess ratio has with the CV of the loss multipliers and simply stated that the dispersion or variance would be lower for the uniform distribution without offering any explanation or support as to why that would be the case.

\*\*\*\*\*

### **Question 13:**

#### ***Model Solution 1***

a:

1. Separate primary losses from excess losses – Total loss distributions can have a very heavy (and therefore unstable) tail. Using separate distribution for primary vs excess loss mitigates this issue because the tails of both distributions will be less heavy.
2. Give greater weight to actual experience for larger risks – The larger the risk, the more stable their expected losses are. If actual experience for small risks was given the same weight as for large risks, then the mods for small risks would be erratic. (I'm assuming that the question is suggesting that terrorism insurance would have same experience component.)

b:

1. Due to the often catastrophic nature of terrorism losses, it is unlikely that primary losses would have much meaning. (In WC, primary losses are a proxy for frequency.)
2. Due to the rarity and catastrophic nature of terrorism losses, it is unlikely that larger risks would have more stable losses than smaller risks.

#### ***Model Solution 2***

a:

1. One proposal is to introduce a limit to how much of any one loss enters ratemaking. Workers compensation has limits for single claimant & multiple claimant accidents.
2. Break losses into primary & excess components with primary losses receiving far more credibility than excess losses. Since primary losses are more stable, this adds stability to the ratemaking.

b:

1. The problem with A1 is that WC experience rating and terrorism ratemaking have different goals. WC experience rating uses past losses to the extent that they are predictive of the future loss to improve individual risk equity. Large losses are capped because to some extent they are bad luck and should not be allowed to swing the mod past a certain point. Terrorism ratemaking is not interested in individual risk equity as much as spreading the cost of terrorism losses across all insureds. Capping losses may improve stability but may lead to inadequacy.

2. The logic behind separating primary and excess (from one perspective) is that primary reflects frequency, which is thought to be more controllable by the insured. Excess reflects severity. Terrorism losses are essentially all excess losses and are not controllable, so a primary and excess split is not appropriate.

**Examiner's Comments:**

\*\*\*\*\*

**Part a**

Most candidates received full credit on this part.

Common responses that garnered full credit included:

- Limit/cap losses that enter into experience rating
- Split losses between primary and excess
- Credibility weight actual experience with expected
- Introduce a ballast value to reduce impact of individual risk experience
- Limit annual rate changes (i.e. swing limits)
- Limit range of mod factors (i.e. max/min mods)
- Vary credibility by size of risk
- Group homogeneous risks
- ... and others

**Part b**

Very few candidates received full credit on this part. In order to obtain full credit, solutions needed to tie to part A and be detailed enough to convey understanding. Partial credit was heavily utilized to differentiate between complete and incomplete responses.

Examples of partial credit responses:

- Limit/Cap – Terrorism losses are rare occurrences and catastrophic, data may not be available due to low frequency nature to accurately put into place an appropriate cap/limit.
- Split – Primary and excess components which would reflect frequency and severity respectively. Terrorism events are extremely rare (low frequency), so a split to reflect frequency would be difficult.
- Credibility – often terrorism attacks focus on major known landmarks. To redistribute this risk to lesser buildings in lesser cities would be unfair.
- Ballast – using a ballast may smooth the mods too much. Resulting in subsidies and not providing an incentive to protect against terrorist (at an individual risk level).
- Swing – in a high terrorism year, swing will be applied to many, many insured's, which may make premiums inadequate.

- Max Mod – For terrorism risks, if it happens again, most likely the modification is representative of the future experience. Since experience rating is used to predict the prospective policy period, it could be argued that the mod factor should not be capped.
- Credibility by size of risk –terrorists tend to strike large targets (world trade, OK city) so it may be impossible to determine what is “credible” experience, especially for small risks unlikely to be targets.

Responses that simply mentioned lack of data or difficulty received zero credit.

Responses had to be related to the response in Part a to receive credit.

\*\*\*\*\*

**Question 14:**

**Model Solution 1**

- 1st year claims made policy effective 1/1/2013
- 3 years of experience used: Occurrence polices effective 1/1/2009 to 12/31/2011 (Rule # 4)

Loss cost = 100,000 \* 70 % = 70,000

	(A)	(B)	(C)	(D)	(E) = (A)*(B)*(C)*(D)	(F)	(G)	(H) = (E)*(F)*(G)
Year	Loss cost	PAF 1 (13B)	PAF 2 (13C)	Detrend	Subject loss cost	EER	LDF (1)	ARULL
Latest (2011 Occ)	70,000	1.33	1.00	0.873	81,276	0.908	0.766	56,530
2 <sup>nd</sup> (2010 Occ)	70,000	1.33	1.00	0.816	75,970	0.908	0.637	43,941
3 <sup>rd</sup> (2009 Occ)	70,000	1.33	1.00	0.763	71,035	0.908	0.528	34,056
<b>Total</b>					<b>228,281</b>			<b>134,527</b>

(1): Losses being evaluated as of 6/30/2012

- 2011 is @ 18 months
- 2010 is @ 30 months
- 2009 is @ 42 months

Table 16 lookup of 228,281

- Z = 0.44
- EER = 0.908
- MSL = 137,350

Mod = Z \* (AER - EER) / EER = 0.44 \* (AER - 0.908) / 0.908 > 0 for debit mod →  
AER > 0.908

AER = (Includable losses + 134,527) / 228,281 → Includable losses > 72,752

Thus only 1 large loss (limited at MSL of 137,350) is sufficient to result in a debit mod so minimum 1 loss.

**Examiner's Comments:**

\*\*\*\*\*

Most candidates were able to complete the appropriate calculations as shown in the table in the model solution. The most common mistakes made by candidates were:

- Selection of PAFs and LDFs based on claims made vs occurrence policies
- Selection of PAFs, Detrend factors and LDFs based on products vs premises/operations
- Selection of LDFs based on the evaluation periods
- Considering that a debit occurs when mod > 1 instead of > 0
- Absence of the demonstration that includable losses are capped by the MSL on 137,350

\*\*\*\*\*



**Question 15:**

**Part a**

**Model Solution 1**

$$\frac{E(x';40M) - E(x';30M)}{E(x; 40M) - E(x;30M)} = \frac{1.08 [E(x;40M/1.08) - E(x;30M/1.08)]}{E(x;40M) - E(x;30M)}$$
$$= \frac{1.08 (4,282,929 - 3,981,081)}{(4,359,735 - 4,063,944)} = 1.102 \Rightarrow + 10.2\%$$

**Examiner's Comments:**

\*\*\*\*\*  
Most candidates had the correct overall thought process and formula. Some common errors were the use of the incorrect limits and omission of the overall trend factor. Some candidates chose a more complex formula using ILFs, but few made the appropriate adjustments to the base limit to achieve the correct answer.  
\*\*\*\*\*

**Part b**

**Model Solution 1**

Insurer may not agree with the lognormal dist. of losses.

If insurer assumes a heavier tail than lognormal dist., more loss would be in excess layer, lightening the impact of excess inflation.

Insurer must have assumed a heavier tail.

**Model Solution 2**

The insurer may have different assumptions regarding the average ground up severity or distribution that it follows. Say the average severity was higher, this would essentially act like an inflation factor shifting more of the excess to higher limits so the impact of trend on this particular layer would be lessened. The shape of the loss distribution could also affect trend in this layer.

**Examiner's Comments:**

\*\*\*\*\*  
Many candidates only listed one or more reasonable differences in assumption, but most candidates lacked a discussion or description of how the insurer's assumption would impact the average increase in the layer.

One alternative answer that some candidates provided was that the insurer assumed the inflation varied by size of loss but was still 8% in total. While this was

an acceptable difference in assumption, no candidate response provided enough detail to demonstrate sufficient comprehension of the learning objective. Therefore, no response related to varying inflation received full credit.

In situations where more than one assumption difference was provided, only the first response was taken into consideration for grading.

\*\*\*\*\*

**Question 16:**

**Model Solution 1**

(1)	(2)	(3)=(2) x exp mod factor	(4)	(5)=(4)/(2)	(6)=(4)/(3)
Policy	Manual Premium	Standard Premium	Losses	Manual LR	Std LR
A	40,000	40,000(.97)=38,800	39,000	0.975	1.0052
B	10,000	10,000(1.4)=14,000	14,500	1.450	1.0357
C	25,000	25,000(.95)=23,750	23,500	0.940	0.9895
D	15,000	15,000(1.33)=19,950	20,500	1.367	1.0276
E	45,000	45,000(.81)=36,450	33,000	0.733	0.9053
<b>Total</b>	<b>135,000</b>	<b>132,950</b>			

a) The statement is somewhat valid, since these two risks have the highest standard loss ratios. However, the standard loss ratios aren't that much higher than the other policies. It's also possible that B and D are poor fits for their current classifications, so their mods might be under or overstated.

b) Off-balance=SP/MP=132,950/135,000=.9848

c)

Policy	Mod	Manual LR	Std LR
E	.81	0.7333	.9053
C	.95	.94	.9895
A	.97	0.975	1.0052
D	1.33	1.367	1.0276
B	1.40	1.45	1.0357

→ No, since the standard LR shows increasing trend as the mods increase, it means the plan is not responsive enough and does not demonstrate premium equity.

**Model Solution 2**

a) Debit mod doesn't mean risk is a bad risk. If experience rating plan is designed appropriate, debit and credit risks are equally desirable. Could just mean risk is a poor fit to manual classification but has good safety program.

Lastly, any loss is mostly pure chance.

b)  $\text{Off-balance} = \text{standard premium} / \text{manual premium} = 132.95 / 135 = .985$

c)

Policy	Mod	Manual LR	Std LR
E	.81	.73	.905
C	.95	.94	.989
A	.97	0.975	1.005
D	1.33	1.367	1.028
B	1.40	1.45	1.036

Equity not achieved → credit risks performing better than debit risks as can be seen by increasing trend in the standard LR. Plan doesn't apply enough credibility

**Model Solution 3, for part c only**

Policy	Mod	Manual LR	Std LR
E	.81	.733	.905
C	.95	.94	.989
A	.97	0.975	1.005
D	1.33	1.36	1.028
B	1.40	1.45	1.036
	E(LR)	1.0916	0.9926
	E(LR <sup>2</sup> )	1.265	0.9875
	V(LR)	0.0734	0.0022

Manual LR widely spread, positive trend → identifies risk

Std LR close to unity (except for mod 0.81), but has positive trend → not responsive enough

Test statistic =  $0.0022 / 0.0734 = 0.03$ . Low test statistic is good.

**Examiner's Comments:**

\*\*\*\*\*

Candidates took two approaches to this question – qualitative and quantitative.

For the qualitative approach, full credit was given for 1) recognizing that the Underwriter was targeting policies B and D as undesirable because of their debit mods, and 2) stating that debit mods don't necessarily mean the risk is undesirable, rather the mod could mean that the risk was a poor fit in their classification or, if the

debit mod was due to poor loss performance, that any single loss could be a matter of pure chance.

For a quantitative approach, partial credit was given for calculating the Standard Loss Ratios for risks B and D and comparing those Loss Ratios to the rest of the risks. To get full credit, though, the candidate still needed to mention that the debit mod could mean that the risk was a poor fit in their classification, or if the debit mod was due to poor loss performance that any single loss could be a matter of pure chance.

About half the candidates received full credit on this part of the question.

#### Part b

For full credit, the candidate needed to produce the correct numerical answer. Most candidates did receive full credit on this part. Partial credit was given for calculating total Manual Premium, total Standard Premium, setting up the off-balance formula correctly, and correctly calculating the ratio based on the calculated Standard/Manual Premium results.

#### Part c

Most candidates correctly calculated the Standard Loss Ratios for all five policies. Additional credit was given for ranking risks by their mod and for recognizing the trend in Standard Loss Ratios in relation to mod and further recognizing that this trend is a result of too little credibility being assigned to the plan. Candidates could also receive partial credit by calculating a test statistic based on the variances of Standard and Manual Loss Ratios.

Some candidates split risks into only two groups (credit risks and debit risks), and calculated a Standard Loss Ratio for each group. These candidates received only partial credit.

\*\*\*\*\*

### **Question 17:**

#### ***Model Solution 1***

- a) i. Using the observed table: 0.42
- ii. avg = 100K  
entry ratio =  $(300 - 200) / 100 = 1$   
xs ratio = lookup(observed) x lookup(excess) =  $0.11 \times 0.4 = 0.044$
- b) 1. Actual experience is used for the lower layers, where it is credible.
2. Fitted ratios are used for higher ratios, where experience may not be credible.
3. The mixed pareto-exponential curve is structured so that it smoothly joins the experience curves at the cutoff point, yielding a result that is consistent.
- c) This is not unreasonable as long as the shapes of the truncated and shifted data above the cutoff point are similar. Can't tell whether it would be appropriate for the dataset w/ the info given.

#### ***Model Solution 2***

- a) i. HG A:  $R_A(50K) = 0.42$  (use observed for  $L < 200K$ )
- ii. HG B:  $(300K - 200K) / 100K = 1$   
 $R(\text{entry ratio } 1) = 0.4$   
 $R_B(300K) = R_{\text{data}}(200K) \times R(1) = 0.11 \times 0.4 = 0.044$
- b) 1. Allows use of empirical data for lower limits, where credible data is available.
2. Curve fitting allows excess ratios to be calculated at higher limits, where data is sparse.
3. Using a mixture allows model to follow exponential decay for medium size losses, but still reflect large tail (pareto has large tail).
- c) It would be reasonable if the shapes of the distributions are similar for the different hazard groups after truncation and normalization.

**Examiner's Comments:**

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Part a

Most candidates correctly calculated each ratio.

Part b

Most candidates were able to provide at least a few advantages. Advantages needed to be uniquely related to the methodology outlined in the question, but within that context many different advantages were accepted. Many candidates listed as separate advantages that the mixed exponential/pareto fits well at the high end and the low end of the truncated excess loss range; since the method would not be valid unless an appropriate distribution is selected for the whole range, that was accepted as a single advantage.

Part c

This was the part of the question that had the most errors. Many candidates stated reasons why it would not be appropriate to merge data from the hazard groups together, without discussing that the observed mean residual lives were found by Mahler to have similar distributions (and the problem above used a single curve for all hazard groups). Many candidates incorrectly discussed why a single distribution (instead of the pareto/exponential) could not produce a reasonable curve.

\*\*\*\*\*

**Question 18:**

**Model Solution 1**

Avg Loss = E = (7,500,000 + 1,100,000 + 1,400,000) / 10 = 1M

r Charge = % capped losses eliminated by Agg + LER

1.1 (1300-1100+1600-1100+1900-1100) / (10\*1,000,000) + 0.1 = 0.25

1.3 (1600-1300+1900-1300) / (10\*1,000,000) + 0.1 = 0.19

1.6 (1900-1600) / (10\*1,000,000) + 0.1 = 0.13

1.9 0.1

2.1 0.1

Risk	Agg Losses limited at 500K per occurrence	loss eliminated
8	1000+300=1300	0
9	1100+500=1600	100K
10	1200+200+500=1900	900K
		1000K

LER = (1,000,000 / 10) / 1,000,000 = 0.1

**Model Solution 2**

Risk	Total unlimited loss	Lim to 500K per occurrence	r=1.1	1.3	1.6	1.9
	2.1					
1	300	300	300	300	300	300
	300					
2	400	400	400	400	400	400
	400					
3	500	500	500	500	500	500
	500					
4	600	600	600	600	600	600
	600					
5	700	700	700	700	700	700
	700					
6	800	800	800	800	800	800
	800					
7	900	900	900	900	900	900
	900					
8	1300	1300	1300	1300	1300	1300
	1300					
9	1700	1600	1100	1300	1600	1600
	1600					
10	2800	1900	1100	1300	1600	1900
	1900					

Average unlimited = 1,000 Average limited = 900

K = 0.1

So charges including per occurrence are:

r charge



1.1  $(1-7500/9000)(1-0.1)+.1 = 0.25$  (modified, see below)

1.3 0.19

1.6 0.13

1.9 0.1

2.1 0.1

$[1-(\text{total limited to } r \text{ \& } 500\text{K per occ}/\text{total limited to } 500\text{K per occ})](1-k)+k$

**Examiner's Comments:**

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Full credit was given for candidates who got all of the correct charges for each entry ratio.

Common errors that resulted in partial credit:

- Multiplying the percent eliminated by 1-LER when the denominator was total losses rather than limited losses (Skurnick Page 127 states that adjustment factor of 1-k is needed only when the charge is represented as a ratio to expected limited losses)
- Limiting losses to 200K rather than 500K resulted in an LER of 0.17 rather than the correct 0.10
- Using total losses of 7.5M instead of 10M (candidates did not add in the columns for the individual accidents greater than 200K). This resulted in an LER of 0.1333.
- Some candidates for entry ratios of 1.9 and 2.1 calculated an erroneous charge rather than just using the LER

\*\*\*\*\*

**Question 19:**

**Part a**

**Model Solution 1**

Retro Premium Before Max/Min = Tax Factor (Basic Premium + Converted Losses + Excess Loss Premium + Retro Development Premium) =  $1.05 * (360,000 + 1,650,000 + 330,000 + 154,000) = 2,618,700$ . Premium within Min/Max so no further adjustments.

$$\text{Basic Premium} = e - (c - 1.0) * E + CI = 400,000 - 0.1 * 1,400,000 + 100,000 = 360,000.$$

$$\text{Converted Losses} = c * \text{Actual limited loss} = 1.10 * 1,500,000 = 1,650,000.$$

$$\text{Excess Loss Premium} = c * SP * ELF = 1.1 * 2,000,000 * 0.15 = 330,000.$$

$$\text{RDP} = c * SP * RDF = 1.1 * 2,000,000 * 0.07 = 154,000.$$

**Model Solution 2**

$$\text{Basic Premium} = 0.2 - 0.1 * 0.7 + 0.05 = 0.18$$

$$\text{Converted Losses} = 1.10 * 0.75 = 0.825$$

$$\text{Excess Loss Premium} = 1.1 * 0.15 = 0.165$$

$$\text{RDP} = 1.1 * 0.07 = 0.077$$

$$\begin{aligned} \text{Retro Premium} &= 2,000,000 * 1.05 * (0.18 + 0.825 + 0.165 + 0.077) \\ &= 2,000,000 * 1.30935 = 2,618,700 \end{aligned}$$

Premium within Min/Max so no further adjustments.

**Examiner's Comments:**

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Typical mistakes on Part A were for missing some part of any of the retro premium components. The following is a list of many of the common errors:

- Missing one of the 3 components that went into the basic premium.
- Missing the ELP or RDP.
- Not using standard premium in the RDP or ELP calculations.
- Not using the LCF in the RDP, ELP, or converted loss.
- Not using limited loss in the converted loss component.
- Forgetting to include the tax multiplier in the final calculation.

- Having all the correct components but the final number calculated incorrectly.
- Incorrect calculations above the max or below the min and where the candidates didn't cap the retro premium appropriately.

Some candidates attempted to calculate the premium at the first adjustment and at the second adjustment and then subtracted premium at the first adjustment from the second adjustment. Although not enough information was given in the question to calculate the premium at the first adjustment, candidates did not have any points deducted for attempting to calculate the premium at this adjustment and then subtracting this premium from the calculated premium at the second adjustment

\*\*\*\*\*

**Part b**  
**Model Solution 1**

Include this because it stabilizes the premium adjustments bringing them closer to ultimate so that the premium paid upfront is closer to final which reduces credit risk of collecting subsequent premiums from adjustments.

**Model Solution 2**

Since losses develop over time, without a RDF, usually losses will be lower at early adjustments which will cause a refund to the insured. Once losses develop higher, the insured will pay the money back. To avoid volatility and back-and-forth payments, retro development factors are used to stabilize premium adjustments.

**Examiner's Comments:**

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Full credit was given to candidates for demonstrating complete understanding of RDF's writing out that that an RDF minimizes the back-and-forth between an insured and the insurer as without RDF's there would typically be a refund early on and then following that up with one of the following additional points of credit risk, investment income, or a mention of losses developing to ultimate leading to the insured having to repay the insurer.

Partial credit was given to candidates that gave a good explanation of 'stabilizing premium' but didn't give a complete description. Candidates that just mentioned how it minimized the back and forth payments as typically there would be a refund but then didn't explain why this would be the typical pattern (i.e. no mention of loss development) or didn't add on why this was beneficial to an insurance company (i.e. credit risk, generating investment income) were only given partial credit.

Deductions were taken for words repeated from the NCCI handout without any explanation which would demonstrate understanding of the material. For example,

responses such as “to account for any future increase in loss cost”, ‘to stabilize premium adjustments’, or “to account for future loss development” were not given credit.

\*\*\*\*\*

## Question 20:

### Model Solution 1

$$E = 1500000 \quad \text{HG Differential} = 1.25$$

$$\text{ELPPF} = 0.24$$

$$\hat{E} = 1140000$$

$$r = 2.5$$

$$\begin{aligned} \text{LUGS} &= 1500000 * 1.25 * \frac{1+0.8*0.24}{(1-0.24)} \\ &= 2940789 \end{aligned}$$

$$\text{ELG} = 28$$

$$X_G = 0.063$$

$$\begin{aligned} \text{Expected loss} + \text{ALAE} &= 360000 + 1140000 * 0.063 \\ &= 431820 \end{aligned}$$

### Model Solution 2

$$\text{Alaska SP} = 2M$$

$$\text{ded} = 225000$$

$$\text{agg ded} = 2850000$$

$$\text{HG} = B$$

$$E = 0.75$$

1. Find ELAPP factors p. RR2

$$\text{ELAPP}(\text{hg}=B, \text{ded}=225000) = 0.24 = F/E$$

2. Find ELG

$$\text{LUGS} = 2000000 * m(s/n) * 0.75 * \frac{1+0.8*0.24}{(1-0.24)}$$

$$m(s/n) \text{ Alaska, HG} = B = 1.25 \leftarrow \text{table lookup}$$

$$= 2000000 * 1.25 * 0.75 * \frac{1+0.8*0.24}{(1-0.24)}$$

$$= 2940789.47$$

$$\rightarrow \text{ELG E} = 2940789 = 28 \text{ (table 2008)}$$

3. Insurance charge:

$$r = \frac{\text{agg ded}}{\text{Limited Losses}} = \frac{2850000}{2M * 0.75 * (1-.24)} = \frac{2850000}{1140000} = 2.5$$

$$\Phi(r=2.5, \text{ELG}=28) = 0.063 \text{ table lookup}$$

4. total expected loss + ALAE = excess losses +  $\Phi$  \* limited losses

$$= (2M * 0.75 * 0.24) + 0.063 * 1140000$$

$$= 360000 + 0.063 * 1140000$$

$$= 431820\$$$

**Examiner's Comments:**

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Common mistakes in this question were:

- Misinterpreting the ELPPF as an ELF. The ELPPF is a percent of losses, the ELF is a percent of premium
- Using the improper table(s). Based on the effective dates listed for Alaska, the 2008 table of expected loss ranges and the 1998 table of insurance charges are the most recent available. Alternate assumptions are acceptable only if an assumption of a different year was stated
- Applying the insurance charge factor to something other than limited expected losses

\*\*\*\*\*

## **Question 21:**

### ***Model Solution 1***

A)  $ELR = 0.8$ , Entry ratio = 1.125  
 $LR = 1.125 \cdot (0.80) = 0.90$

Insurance Savings @ entry ratio 1.125 (before standardizing)  
 $= 0.05(0.9-0.2) + (0.2-0.05)(0.9-0.4) + (0.55-0.2)(0.9-0.6) + (0.7-0.55)(0.9-0.8) = 0.23$

After standardizing  $\rightarrow 0.23/0.80 = 0.2875$

B) Since the average loss ratio increases, entry ratio of 1.125 would correspond to a higher loss ratio, hence increasing the insurance savings.

### ***Model Solution 2***

A)  $R = 1.125 = L/80\% \rightarrow L = 90\%$

Loss Ratio of 180%  $\rightarrow R = 2.25$

Loss Ratio of 120%  $\rightarrow R = 1.5$

Loss Ratio of 100%  $\rightarrow R = 1.25$

$\Phi(1.5) = (10\%)(2.25 - 1.5) = 0.075$

$\Phi(1.25) = (20\%)(1.5 - 1.25) + \Phi(1.5) = 0.05 + 0.075 = 0.125$

$\Phi(1.125) = (30\%)(1.25 - 1.125) + \Phi(1.25) = 0.0375 + 0.125 = 0.1625$

$\Psi(1.125) = \Phi(1.125) + R - 1 = 0.1625 + 1.125 - 1 = 0.2875$

B) Since  $\Psi = \Phi + R - 1$ , then the savings will increase because an increase to the highest decile will increase the charge, all else being equal. The average loss ratio will also increase, making  $R = 1.125$  correspond to a higher loss ratio. This increases the area between  $R$  and  $F(x)$ , further increasing the savings.

**Examiner's Comments:**

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Part a:

There were 2 principal methods to calculate the insurance savings. The candidate had the option to either calculate the savings directly, or to first calculate the insurance charge and then use the relationship between the insurance charge and savings to produce the correct solution.

Using the former method, the candidate would need to determine the area above the curve  $F(x)$  and below the loss ratio. The loss ratio is calculated as the product of the expected loss ratio (80%) and the entry ratio (1.125), or 90%. The area is equal to 0.23. The candidate must then standardize the savings by dividing the area by the expected loss ratio of 80%, producing the insurance savings of 0.2875 or 28.75%.

Using the latter method, the candidate would need to first calculate the insurance charge, which is the area below the curve  $F(x)$  and above the loss ratio of 90% (as described above). This area is equal to 0.13. To calculate the insurance charge, the candidate must standardize by dividing the area by 0.80, producing the insurance charge of 0.1625 or 16.25%. The savings is derived by using the formula:

$$\Psi = \Phi + r - 1 = 0.1625 + 1.125 - 1 = 0.2875 \text{ or } 28.75\%.$$

Common errors made by candidates included:

- Failing to standardize the savings (or the charge) by the 80% loss ratio;
- Providing only the insurance charge, rather than the savings;
- Not calculating the area between the curve and the loss ratio correctly.

Part b:

Candidates received full credit if both of the following were provided in the solution:

- The direction that the insurance savings would move; and
- The correct rationale for this change.

One potential solution would explain that because the 90<sup>th</sup> percentile corresponds to a 120% loss ratio, which is higher than the 1.125 entry ratio, there will be an increase to the insurance charge. Referencing the formula  $\Psi = \Phi + r - 1$ , the candidate can demonstrate that an increase to the charge would imply a simultaneous increase in the savings, all else being equal.

Alternatively, the candidate could have described that an increase in the average loss ratio would imply a higher loss ratio at the entry ratio of 1.125. This would in turn increase the area below the curve  $F(x)$  and above the loss ratio, implying an increase in savings.

\*\*\*\*\*



**Question 22:**

**Part a**

**Model Solution 1**

Total Losses = 182,600

$$\frac{21(100) + 50(250) + 500 \times [42 + 37 + 22]}{182,600} = 0.3565$$

**Model Solution 2**

$$21(100) + 50(250) + 500 \times [42 + 37 + 22] = 65,100$$

$$21(100) + 50(250) + 42(500) + 37(1,000) + 22(5,000) = 182,600$$

$$LER = \frac{65,100}{182,600} = 0.3565$$

**Model Solution 3**

$$\begin{aligned} LER &= \frac{\text{Losses Eliminated}}{\text{Total Losses}} \\ &= \frac{21(100) + 50(250) + 500 \times [42 + 37 + 22]}{21(100) + 50(250) + 42(500) + 37(1,000) + 22(5,000)} \\ &= \frac{65,100}{182,600} = 35.65\% \end{aligned}$$

**Examiner's Comments:**

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A large majority of candidates received full credit for this question. Most deductions were due solely to simple arithmetic errors. However, there were several candidates that used an incorrect formula (typically something equivalent to 1-LER) which resulted in less credit being received.

A small subset of candidates used an average severity approach and this was an acceptable approach to solve the problem.

\*\*\*\*\*

**Part b**  
**Model Solution 1**

$$\text{Total Losses} = 21(110) + 50(275) + 42(550) + 37(1,100) + 22(5,500) = 200,860$$

$$\frac{21(110) + 50(275) + 500 \times [42 + 37 + 22]}{200,860} = 0.3314$$

$$\frac{0.3314 - 0.3565}{0.3565} = -0.07 \text{ or } -7\%$$

**Model Solution 2**

$$\text{New Total} = 182,600 \times 1.1 = 200,860$$

$$\text{New Ded} = 1.1 \left[ 100(21) + 250(50) + \frac{500}{1.1}(42 + 37 + 22) \right] = 66,560$$

$$\text{New LER} = 0.3314$$

$$\% \text{ Change in LER due to inflation} = \frac{0.3315 - 0.3565}{0.3565} = -7.04\%$$

**Model Solution 3**

Number of Claims	Trended @ 10% (Severity)
21	110
50	275
42	550
37	1,100
22	5,500

$$\text{LER} = \frac{110 \times 21 + 275 \times 50 + 500(42 + 37 + 22)}{182,600 \times 1.1}$$

$$= \frac{66,560}{200,860} = 33.14\%$$

$$\% \text{ Change in LER} = 33.14\% - 35.65\% = -2.51\%$$

**Examiner's Comments:**

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Similar to Part a, a large majority of candidates correctly calculated the trend-adjusted LER. However, a good number of candidates stopped there, leaving out the final percent change in LERs that the question asked for. This was by far the most common deduction. It should also be noted that if a candidate got the wrong answer for this question due to an error in Part a, but everything else was done correctly, it was still possible to receive full credit for this part.

A small subset of candidates used an average severity approach and this was approach was an acceptable way to solve the problem.

The final answer was sensitive to the number of decimal places used in interim calculations. Credit was given regardless of the number decimals the candidate chose to use.

\*\*\*\*\*

**Part c**  
**Model Solution 1**

LC for a straight deductible policy can increase more, because the policy is charged on excess of deductible losses. The trend on the excess, tend to increase more than the aggregate trend bc losses that have historically not pierced the layer will now, due to the trend.

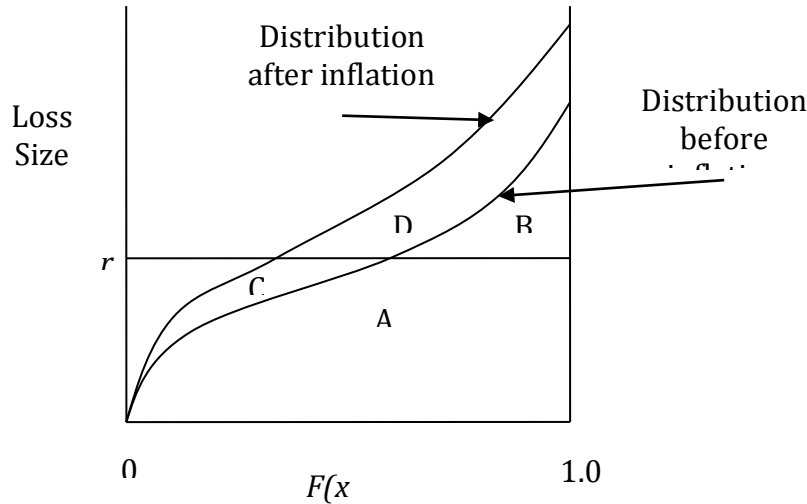
**Model Solution 2**

Each loss that was previously over the deductible increases by the trend, but after subtracting the unchanging deductible, the % change in the net loss is greater than the trend selection

$$\left. \begin{array}{l} \text{Example: } 1,000 \times 1.1 - 500 = 600 \\ 1000 - 500 = 500 \end{array} \right\} 20\% \text{ increase for a } 10\% \text{ trend}$$

Also, new losses pierce the deductible, further increasing total losses after inflation

**Model Solution 3**



As can be seen graphically, when losses increase due to inflation, excess losses increases more than total losses:

$$\frac{D+B}{B} > \frac{A+B+C+D}{A+B}$$

**Examiner's Comments:**

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The results of this question were fairly mixed. There were several ways to get full credit here, and while most candidates received some credit, it was common to not receive full credit because of an overly brief statement that didn't fully explain why the deductible policy's loss cost could increase by more than ground-up trend.

Some candidates used a Lee diagram to show the impact of the trend on the loss cost (area above the deductible). When this diagram was accurately constructed and labeled, this approach received full credit.

Simply stating that some claims would pierce the deductible was not enough for full credit. The candidates needed to add a statement with respect to how this affected total losses in the insured layer (loss cost) or something to that effect.

Some candidates mentioned that the number of claims near the deductible, number of claims above the deductible, or some other count, explained why the loss cost could increase by more than the ground-up severity trend. The relative number of

these claims is not the reason for this phenomenon. Credit was not deducted for this statement, but it in of itself, was not enough for full credit.

\*\*\*\*\*

### **Question 23:**

#### **Model Solution 1**

$$E = 100,000; G = 200,000; H = 50,000; e = 20,000; c = 1.35$$

$$b = e - E(c - 1) + cI$$

$$= 20,000 - 100,000(0.35) + 1.35 * [100,000 (\Phi(r_G) - \Psi(r_H))]$$

$$r_G = \text{Loss}_G / E = 200,000 / 100,000 = 2$$

$$r_H = \text{Loss}_H / E = 50,000 / 100,000 = 0.5$$

$$\Phi(2) = 0.49$$

$$\Psi(0.5) = \Psi(0.5) + 0.5 - 1 = 0.76 + 0.5 - 1 = 0.26$$

$$b = 20,000 - 35,000 + 135,000 (0.49 - 0.26) = 16,050$$

#### **Model Solution 2**

$$b = e - (c-1)E + c(X_g - S_h)E$$

$$E = 100,000$$

$$e = 20,000$$

$$r_{H \text{ min loss}} = 50K / 100K = 0.5$$

$$X_H = 0.76$$

$$S_h = X_h + r_H - 1 = 0.76 + 0.5 -$$

$$1 = 0.26$$

$$r_{G \text{ max loss}} = 100K / 100K = 2$$

$$X_G = 0.40$$

$$b = 20,000 - (1.35 - 1)100,000 + 1.35(.49 - .26)100,000$$

$$= \$16,050$$

#### **Model Solution 3**

$$\text{Insurance charge} = E [\Phi(r_G) - \Psi(r_H)]$$

$$= 100,000 [\Phi(200,000/100,000) - \Psi(50,000/100,000)]$$

$$= 100,000 [\Phi(2) - \Psi(0.5)]$$

$$= 100,000 [\Phi(2) - (0.5 + \Phi(0.5) - 1)]$$

$$= 100,000 [0.49 - 0.26] = 23,000$$

$$\text{Converted Insurance Charge} = c * \text{Ins Charge} = 1.35(23,000) = 31,050$$

$$\text{Basic Premium} = \text{Expense \& Profit Provision} - (c-1)\text{Expected Loss} +$$

$$\text{Converted Ins Charge}$$

$$= 20,000 - (1.35-1)(100,000) + 31,050$$

$$= 16,050$$

#### **Model Solution 4**

$$R = (E + e) T$$

$$R = (b + cL) T = (B + crE) T$$

$$E = 100,000$$

$$L_H = 50,000$$

$$L_G = 200,000$$

$$r_{GE} = L_G \Rightarrow r_G = L_G / E = 200,000 / 100,000 = 2 \Rightarrow X_G = 0.49$$

$$r_H = 50,000/100,000 = 0.5 \Rightarrow X_H = 0.76$$

$$X_H - X_G = [(e + E) - H/T] / cE \Rightarrow (0.76 - 0.49) = [(20 + 100) - H/T]/(1.35*100) \Rightarrow H/T = 83.55$$

$$r_G - r_H = (G - H)/cET \Rightarrow (2 - 0.5) = [G/T - 83.55]/(1.35*100) \Rightarrow G/T = 286.05 \text{ (this line included but not needed)}$$

$$H/T = 83.55 = (b + c*50) \Rightarrow b = 16.05$$

$$B = 16,050$$

**Examiner’s Comments:**

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Most candidates got full credit on this problem. Common mistakes included:

- Arithmetic errors
- Looking up the wrong value in the sample Table M, or copying a charge to where a savings belonged, or similar errors.
- Using the savings at the minimum for the insurance charge, instead of the charge-at-max minus the savings-at-min.
- Using only the charge-at-max for the insurance charge.
- Neglecting to multiply the insurance charge by the loss conversion factor.
- Adding, rather than subtracting, the (c-1)E term.
- Although “e”, the expense and profit provision, was given in the statement of the problem as \$20,000, some candidates assumed that e – (c-1)E was \$20,000. Some may have been confused that (c-1)E was larger than \$20,000, because it is unusual for the variable expense in the premium calculation to be greater than the total expense + profit provision, since the variable expense is often just the claims-related portion of the expense. However, this can happen either if there is enough expected investment income on the reserves (a negative expense included in the “profit provision”) to offset the underwriting expenses, or if the policy-holder and the underwriter agree to load some of the fixed expenses into the loss conversion factor.
- Failing to see how to put the pieces together to solve the problem.

Some candidates commented on the lack of a tax load in this problem, but they either proceeded ignoring the taxes, or assumed a tax rate (usually no taxes, or a factor of 1.0) and no one lost credit based on this.

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