

Exam LC



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Exam LC

Models for Life Contingencies

1.5 HOURS

May 9, 2016

INSTRUCTIONS TO CANDIDATES

- This 30 point examination consists of 15 multiple choice questions worth 2 points.
- To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.
 - Fill in that it is Spring 2016 and that the exam name is LC.
 - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
 - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
 - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.
- Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.
 - Verify that you have a copy of “Tables for CAS Exam LC” included in your exam packet.
5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.
6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.
7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.
8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a copy of the examination from the CAS Web Site.

All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.
9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.
10. The exam survey is available on the CAS Web Site in the “Admissions/Exams” section. Please submit your survey by May 23, 2016.

END OF INSTRUCTIONS

1.

You are given the following information regarding a survival model:

- Survival follows an exponential distribution.
- The cumulative hazard rate function, $\Lambda_0(t) = 0.32$
- $e_0^{\circ} = 50$

Calculate t .

- A. Less than 18
- B. At least 18, but less than 20
- C. At least 20, but less than 22
- D. At least 22, but less than 24
- E. At least 24

2.

An insurance company writes 1,000 policies on Jan 1st, 2015, and another 1,100 on Jan 1st, 2016. No other policies are written.

Assume:

- Policies remain effective unless canceled.
- Policies can cancel anytime.
- Policy lifetimes are modeled by the distribution:

$$F(x) = 1 - e^{-\left(\frac{x}{\lambda}\right)^k}, \quad \text{where } x \geq 0, \quad k = 2, \quad \lambda = 4$$

- x is the policy lifetime, measured in years.

How many policies are expected to be in effect on Jan 1st, 2017?

- A. Less than 1,750
- B. At least 1,750 but less than 1,800
- C. At least 1,800 but less than 1,850
- D. At least 1,850 but less than 1,900
- E. At least 1,900

3.

You are given the following information:

- $\mu_x = \frac{2}{x+1} + \frac{2}{80-x}$ for $0 \leq x \leq 80$
- $l_0 = 10,000$

Calculate the expected number of failures between ages 1 and 5.

- A. Less than 2,000
- B. At least 2,000 but less than 2,100
- C. At least 2,100 but less than 2,200
- D. At least 2,200 but less than 2,300
- E. At least 2,300

4.

You are given the following information:

- Deaths are uniformly distributed over each year of age.
- $l_x = 10,500$
- $l_{x+0.3} = 10,185$
- $d_{x+1} = 1,134$

Calculate ${}_{0.25|1.5}q_x$.

- A. Less than 0.120
- B. At least 0.120, but less than 0.135
- C. At least 0.135, but less than 0.150
- D. At least 0.150, but less than 0.165
- E. At least 0.165

5.

A machine currently has a single engine whose time to failure has a constant force of mortality, μ .

The machine fails when it no longer has an operating engine.

You are given two options to extend the time to failure of the machine.

1. Upgrade the single engine, halving the force of mortality.
2. Add a second engine operating in parallel, whose time to failure is independent of the first engine, and has a constant force of mortality, $k\mu$. The machine fails when both engines fail.

You find that the machine's expected time to failure is the same under each of the two options.

Calculate k .

- A. Less than 0.2
- B. At least 0.2, but less than 0.4
- C. At least 0.4, but less than 0.6
- D. At least 0.6, but less than 0.8
- E. At least 0.8

6.

You are given the following:

- (x) and (y) are subject to the same force of mortality other than their smoking status.
- (x) is a 55-year-old smoker.
- (y) is a 55-year-old who no longer smokes.
- Quitting smoking reduces the force of mortality by one-third.
- The time-to-death random variables T_x and T_y are independent.
- The force of mortality for (x) is given as:

$$\mu_{x+t} = \frac{0.6}{45-t} \text{ for } 0 < t < 45$$

Calculate $\text{Var}(T_{xy})$.

- A. Less than 100
- B. At least 100, but less than 120
- C. At least 120, but less than 140
- D. At least 140, but less than 160
- E. At least 160

7.

You are given the following information about Jake, a 45-year-old actuarial science professor:

- His career is subject to 2 decrements.
- Decrement 1 is mortality, which follows the uniform distribution with $\omega = 110$
- Decrement 2 is leaving academic employment, with $\mu_{45}^{(2)}(t) = 0.08$, for $t \geq 0$

Calculate the probability that Jake remains an actuarial science professor for at least 10 years, but less than 15 years.

- A. Less than 0.125
- B. At least 0.125, but less than 0.135
- C. At least 0.135, but less than 0.145
- D. At least 0.145, but less than 0.155
- E. At least 0.155

8.

For a discrete triple decrement model, you are given the following information:

- $\mu_{x+t}^{(1)} = \mu$
- $\mu_{x+t}^{(2)} = 2\mu$
- $\mu_{x+t}^{(3)} = \mu + 0.03$
- ${}_tP_x^{(r)} = e^{-0.27t}$

Calculate ${}_{5|3}q_x^{(3)}$.

- A. Less than 0.045
- B. At least 0.045, but less than 0.055
- C. At least 0.055, but less than 0.065
- D. At least 0.065, but less than 0.075
- E. At least 0.075

9.

Mike just turned 27 and is currently single.

Assume the following:

- Q is the transition matrix between the states Single (S), Dating (D), and Married (M).
- Q applies to people between ages 25 and 40.

$$Q = \begin{array}{c} S \quad D \quad M \\ \begin{array}{l} S \\ D \\ M \end{array} \begin{bmatrix} .60 & .39 & .01 \\ .45 & .45 & .10 \\ .05 & .05 & .90 \end{bmatrix} \end{array}$$

What is the probability that Mike will be married at the age of 30 (i.e. after 3 transitions)?

- A. Less than 7%
- B. At least 7%, but less than 8%
- C. At least 8%, but less than 9%
- D. At least 9%, but less than 10%
- E. At least 10%

10.

The survival distribution of a life (x) is modeled using a homogeneous Markov process with two states: Living and Deceased.

Transitions between states occur at the end of each period, and once the life is Deceased, it remains Deceased.

The median future lifetime of (x) is 30 years.

Calculate the transition probability from Living to Deceased.

- A. Less than 0.01
- B. At least 0.01, but less than 0.03
- C. At least 0.03, but less than 0.05
- D. At least 0.05, but less than 0.07
- E. At least 0.07

11.

You are given the following information:

- Credentialed actuaries at your company earn \$30,000 per year more than analysts.
- Salary is paid continuously.
- The time until you earn your credentials is distributed exponentially with $\mu = 1/7$
- You are an analyst, and you take exams continuously until you earn your credentials.
- You will work for your current employer for the next 40 years until you retire, at which point you will also stop taking exams if you are not yet credentialed.
- However, you would quit taking exams permanently (but remain an analyst) if you were given K today, where K is the actuarial present value of the additional lifetime salary you expect to earn with your credentials.
- $\delta = 0.05$

Calculate K .

- A. Less than 325,000
- B. At least 325,000, but less than 350,000
- C. At least 350,000, but less than 375,000
- D. At least 375,000, but less than 400,000
- E. At least 400,000

12.

You are given the following information:

- A special 20-year contract issued to (35) will pay a unit benefit at the end of year of failure if failure occurs during the 20-year period, or will refund the net annual premiums paid if (35) survives to the end of the 20-year period.
- Net annual premiums of P are paid at the beginning of each year for the 20-year period.
- P is calculated according to the equivalence principle.
- $d = 0.06$
- $A_{35:\overline{20}|} = 0.50$
- $A_{35:\overline{20}|}^1 = 0.30$

Calculate the net annual premium, P .

- A. Less than 0.060
- B. At least 0.060, but less than 0.070
- C. At least 0.070, but less than 0.080
- D. At least 0.080, but less than 0.090
- E. At least 0.090

13.

You are given the following information:

- Survival follows the exponential distribution.
- $\bar{A}_x = 0.4333$
- $\bar{a}_x = 6.6667$
- $\bar{a}_{x:\overline{n}|} = 5.7182$

Calculate n .

- A. Less than 11.5
- B. At least 11.5, but less than 12.5
- C. At least 12.5, but less than 13.5
- D. At least 13.5, but less than 14.5
- E. At least 14.5

14.

An auto insurance company sells policies that have a \$300 deductible. The insurer wants to begin offering a diminishing deductible benefit that reduces the deductible by \$100 at the beginning of each year, when there were no claims in the prior year.

You are given the following information:

- The initial deductible amount is \$300.
- Accidents occur at the end of each year, and upon occurrence of an accident, the carrier pays \$1,000 less the deductible amount.
- The deductible amount is reset to \$300 the year after an accident.
- The probability of an accident in a given year is 10%.
- No more than one accident can occur in a year for a given policy.
- Policies are effective for a three-year term.
- $i = 0.06$

Calculate the actuarial present value of the diminishing deductible benefit over the lifetime of a single policy.

- A. Less than \$15
- B. At least \$15, but less than \$17
- C. At least \$17, but less than \$19
- D. At least \$19, but less than \$21
- E. At least \$21

15.

You are given the following information:

- A group of 100 individuals establish a fund that will pay unemployment benefits over the next year.
 - State 1 = Employed.
 - State 2 = Unemployed.
 - State 3 = Retired.
 - Individuals transition between states according to a homogenous Markov chain process:
- $$Q = \begin{vmatrix} 0.90 & 0.05 & 0.05 \\ 0.60 & 0.30 & 0.10 \\ 0.00 & 0.00 & 1.00 \end{vmatrix}$$
- An individual classified as Unemployed at the end of the year receives 15,000 at that time.
 - Individuals transition once per year.
 - $i = 0.06$
 - The initial funding is established at the beginning of the year according to the equivalence principle.

State	Individuals at $t = 0$
1	80
2	20
3	0

Calculate the initial funding per employed individual at time $t = 0$.

- A. Less than 1,400
- B. At least 1,400, but less than 1,500
- C. At least 1,500, but less than 1,600
- D. At least 1,600, but less than 1,700
- E. At least 1,700

Spring 2016 Exam LC Solution Key

1. A

2. C

3. C

4. D

5. D

6. E

7. D

8. B

9. D

10. B

11. C

12. B

13. C

14. E

15. E